Manhattan College

Center for Geotechnology

Geomaterials Research Project
-
A Technical Note re Calculating the Fundamental Period of an EPS-Block-Geofoam Embankment

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by

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former Civil Engineering Department geotechnical engineering program (CE/GE)
research report series are available in PDF format via the Internet at:

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The Manhattan College School of Engineering *Center for Geotechnology* and Its Mission

The **Manhattan College School of Engineering** ***Center for Geotechnology*** **(CGT)** is a unique organization that strives to be more than the typical academic research center or institute. It was founded in 2001 at the personal initiative of Prof. John S. Horvath, Ph.D., P.E. of the Civil Engineering Department who serves as its first Director. The CGT was the result of Prof. Horvath's evolutionary realization after almost 30 years of geotechnical engineering practice that the explosive growth in geotechnical and geoenvironmental engineering technology has made it difficult for the engineering practitioner to keep abreast of new technical developments. The traditional academic approach of simply publishing research results in narrowly disseminated technical reports and papers (a philosophy of 'if you print it, they will learn') has proven to be an increasingly ineffective way of reaching practitioners and moving the state of art to the state of practice. There is an ever-growing gulf between these two states of knowledge in geotechnical engineering. The critical need for a total rethinking of how life-long continuing education is achieved not only for engineering practitioners but academicians themselves is evidenced by the appearance of 'teach-the-teacher' training courses in drilled shaft foundations and geosynthetics beginning in the late 1980s. If even academicians cannot keep up with new developments by reading journal papers and conference proceedings, how can practitioners be expected to? The stagnation of geotechnology also affects current engineering students and perpetuates the cycle. The desirability of involving the practitioner in the process of formulating research programs so that they may have a more direct and immediate benefit to practice is also something that is now recognized more and more.

The CGT seeks to address the current need for effective, meaningful continuing education by recognizing that the cycle of growth for any technology has three interdependent components, what can be called the 'trilogy of technology'. Like a three-legged stool, each of these components must be of equal length and strength if a given technology is to succeed. Thus the CGT has adopted a holistic strategy of supporting geotechnology growth by recognizing the need to concurrently address:

- Technology advancement through research and development that involves not only the engineering practitioner but also other end users of geotechnology such as construction contractors and material manufacturers to the greatest extent practicable.

- Technology transfer (T^2) through education of engineers, contractors and manufacturers in a multi-faceted, proactive way.

- Technology documentation through standards development so that all end users (practitioners, contractors and manufacturers) of a given technology work to a common set of guidelines.

This trilogy-of-technology growth cycle is the cornerstone of all activities of the CGT. It is important to note that the interaction of these three components is never completed but assumes a constant cycle that leads to continuous growth of a technology. This concept is embodied in the CGT logo of three interconnected arrows that is displayed on all CGT documents.

Another issue that the CGT seeks to address is the increasing specialization and compartmentalization within civil engineering. This is a problem that affects numerous professions and can only be expected to worsen as technology grows and a professional must concentrate their education and practice ever more. Despite the inevitability of specialization, there are steps that can be taken to soften its effects by sensitizing and informing professionals of the work of those in allied professions. Toward this end, the CGT seeks to involve other areas of related professional activity such as structural engineering, environmental engineering and science, and geology in both its research and, especially, technology transfer activities.

The CGT receives no direct financial support on a regular basis from Manhattan College. Thus the success and growth of the CGT is totally a function of outside funding from individuals and organizations whose philanthropic philosophies are consistent with the stated goal of the CGT to treat technology growth in a more holistic fashion than is typically done in academia and considers the entire process from research to standards with end-user input at all stages. In addition, as part of its mission to promote technology transfer through education to the greatest extent practicable the CGT is willing to partner with industry and other academic institutions not only in research but also technology transfer and standards activities on any topic relevant to geotechnical or geoenvironmental engineering. The new Manhattan College School of Engineering William J. Scala Academy Room, which is located on the main floor of the Leo Engineering Building and available for CGT activities, offers modern facilities for hosting technology transfer activities. One benefit of Manhattan College's location on the northern edge of New York City adjacent to both Interstate I-87 and mass transit is that it is quite accessible (including free, off-street parking adjacent to Leo Engineering Building) from both within and outside the City. When appropriate, the CGT will bring its technology transfer activities off campus to meet the needs of a particular activity.

Regardless of financial support, the ultimate success and growth of the CGT will depend on its being responsive to the needs of the engineering practitioner. Towards this end, the CGT welcomes input from practitioners on an ongoing, continual basis. Suggestions for future research and technology transfer activities based on perceived needs in practice are always welcome. There is no topic that is too modest or simple for consideration. In fact, much of the research conducted by Prof. Horvath since he came to Manhattan College in 1987 has been based on ideas, large and small, that he developed as a result of his years in engineering practice. Additional information about the CGT as well as access to published documents and other resources can be found on the Internet at

www.engineering.manhattan.edu/civil/CGT.html.

Preface

The ultimate goal of any research in an applied science such as civil engineering should be to extend the state of knowledge so that today's state of art become tomorrow's state of practice. An important component of this continuous cycle of technological growth is the use of newly developed theory in practice. Practical application and, whenever and however possible, verification of calculated results are important aspects of developing users' confidence in new theories.

This technical note has been prepared with these goals in mind. Although block-molded expanded polystyrene (EPS) was developed circa 1950 and has been used as a geofoam material in the functional application of lightweight fill for road construction since at least 1972, much is still being learned about how EPS fills behave under seismic loading. Since May 2000, I have been fortunate to be involved professionally in the quintessential 'Big Dig', the Central Artery/Third Harbor Tunnel (CA/T) project in Boston. My specific involvement has dealt with the use of EPS geofoam for several embankments for the I-90/I-93 interchange in South Boston. What at first appeared to be a technologically routine application of EPS geofoam has actually resulted in pushing the edge of the technological envelope in several aspects, especially with regard to seismic design. This report documents one small but important aspect of that work.

I am deeply indebted to Hany L. Riad, Ph.D., P.E. of the CA/T engineering staff for his unwavering, intense interest in the use of EPS geofoam on this project. He was, from my perspective as the consultant on this project, the 'dream' client to work with, someone who never lost interest in the details and would always ask truly insightful questions that stimulated me to refine, clarify and crystallize my own thinking about numerous aspects of EPS geofoam. In the process, Dr. Riad came up with several new ideas of his own that have been and will be documented in other publication venues. I am also grateful to the Federal Highway Administration and the CA/T owner, the Massachusetts Turnpike Authority, for their interest and support throughout this project. However, I should emphasize in closing that the opinions expressed in this technical note are solely mine and do not necessarily represent those of Dr. Riad or anyone else involved in the CA/T project.

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Background

The Central Artery/Third Harbor Tunnel (CA/T) Project in Boston, Massachusetts, U.S.A. is one of the largest public-works undertakings in American history. Although not without its controversies at many levels and for many reasons, there is no denying the fact that the 'Big Dig', as it is referred to colloquially and universally, has extended the state of civil-engineering knowledge in many ways. Numerous publications have been generated about various technical aspects and achievements of the CA/T since planning and design work began in earnest in the late 1980s.

Of relevance to this report is the use of block-molded expanded polystyrene (EPS-block) geofoam[a] as a lightweight-fill material on Contract C09C2[b] of the CA/T. This contract was one of the last pieces of the CA/T project to go to design and construction. As of the date of this report (February 2004) work on this contract was still in progress.

By the time design work on Contract C09C2 began circa 2000, the use of EPS-block geofoam as lightweight fill was not a new geotechnology although its use in various countries, including the U.S.A., was still somewhat novel to many. In particular, design methodologies and material and construction specifications were far from standardized in the U.S.A. at the time the Contract C09C2 design work began. Although there had been attempts to develop global guidelines for the most-common use of EPS-block-geofoam lightweight fill (for constructing road embankments), these guidelines (*Matériaux* 1997) reflected successful-but-outdated technology and did not include any information related to seismic design. The latter element was an important consideration for the numerous planned embankments that were part of Contract C09C2.

Fortunately, during the same time frame that design work for Contract C09C2 was beginning a government-funded research project was being conducted in the U.S.A. as part of the National Cooperative Highway Research Program (NCHRP) that is administered by the Transportation Research Board, a division of the National Research Council[c]. Major objectives of this research were to develop, for the first time in U.S. practice, comprehensive design guidelines as well as material and construction standards for using EPS-block geofoam as lightweight fill for road embankments on soft ground. It is of interest to note that although the scope of this NCHRP research project was intentionally quite narrow for political reasons, the subsequent outcomes of this research were and still are far more useful in practice. In fact they can be applied to any project where block-molded EPS is used as a geofoam lightweight-fill material.

Due to the coincidental timing and the publicly-funded nature of the CA/T project, the design engineers for the CA/T owner, the Massachusetts Turnpike Authority, were given access to the otherwise-restricted Phase I report for this NCHRP research project (Stark et al. 2000). Thus by happenstance CA/T Contract C09C2 was the first actual project to make use of the comprehensive design- and construction-related guidelines developed as part of the NCHRP research. The material in Stark et al. (2000) was used by the CA/T design engineers to develop a

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[a] For those who are unfamiliar with the distinction of *block-molded* EPS it is relevant to note that it is possible to make EPS by what is called the *shape-molding* process. Although shape-molded-EPS geofoam products are still relatively rare, they are growing in number and use and can be common regionally. The EPS molding process is an important distinction for any load-bearing geofoam product (such as blocks for use as lightweight fill) because the molding process affects the isotropy of the mechanical (stress-strain-time) properties of EPS. Despite this behavioral nuance between block- and shape-molded EPS, note also that in present global usage when only 'EPS' is referred to in geofoam applications the block-molded product is almost always inferred. This is in much the same way that the word 'concrete' invariably means 'portland-cement concrete' which ignores the fact that there are many different kinds of concretes (asphaltic concrete to name but one).

[b] Officially titled "I-93/I-90 Interchange, Ramps and Surface Restoration at Albany Street".

[c] NCHRP Project No. 24-11 for Fiscal Year 1998 titled "Guidelines for Geofoam Applications in Embankment Projects".

comprehensive suite of project-specific documents (analysis-and-design criteria, a detailed design example, and a material-and-construction specification) specifically for the technical and contractual needs of Contract C09C2.

As it turned out, CA/T Contract C09C2 served not only to provide an unusually comprehensive and critical assessment of the preliminary NCHRP research outcomes but even pushed the edge of the technological envelope of EPS-block geofoam. Much of this work has recently been documented in the literature. Riad et al. (2004) contains an overview of the design process for selecting lightweight-fill materials on CA/T Contract C09C2 (EPS-block geofoam was neither the sole material considered nor the sole material eventually used). The overall benefits of using EPS-block geofoam as a lightweight fill for road construction in an urban environment such as encountered in South Boston are presented in Riad et al. (2003a) and the specific technical innovations developed as part of the detailed design for Contract C09C2 are summarized in Riad et al. (2003b)[d]. It is relevant to note that the most significant technical achievements of the Contract C09C2 embankments that incorporated EPS-block geofoam were in the area of seismic design (Riad and Horvath 2004). As it turned out on this project, seismic issues dominated the design of the EPS-block geofoam fills and ultimately dictated material requirements of the EPS itself.

Purpose and Scope

The key element of the seismic analysis or design of any fill that incorporates EPS-block geofoam is to estimate the seismic amplification of the horizontal component of the free-field ground-surface acceleration at the top of the embankment[e]. This can be done in various ways, either approximately using fixed values or empirical formulas given in building or design codes, or more-rigorously using site-specific response spectra. Relevant to the present discussion is that most methodologies require knowledge of the fundamental period[f], T_o, of the embankment.

The purpose of the technical note documented in this report is to review, and in the process critically reassess, the origin and derivation of the following equation that is considered to be the current state-of-art for calculating T_o of an embankment composed primarily of EPS-block geofoam used as a lightweight-fill material. This equation was ultimately incorporated into the CA/T Contract C09C2 Project Design Criteria and was taken from Stark et al. (2000):

$$T_o = 2\pi\left\{\frac{\sigma'_{v_o} H}{E_{t_i} g}\left[4\left(\frac{H}{B}\right)^2 + \left(\frac{12}{5}\right)(1+\nu)\right]\right\}^{0.5} \tag{1a}$$

where:

B = embankment width,

E_{t_i} = initial tangent Young's modulus of the EPS,

g = gravitational constant (32.2 ft/s^2 in imperial units or 9.81 m/s^2 in SI units),

H = embankment height,

ν = Poisson's ratio of the EPS (typically taken to be $\cong$ 0.1 within the elastic range as is applicable for lightweight-fill applications), and

σ'_{v_o} = vertical effective stress acting on the top of the EPS (from dead loads only here).

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[d] A more-accessible reference is Horvath (2003).

[e] Note that the ground-surface motion may itself be greater than the underlying bedrock motion due to soil-amplification effects within the foundation soil. Soil-column amplification is outside the scope of the present discussion but was also a factor for the CA/T Contract C09C2 embankments.

[f] Which is the inverse of the natural frequency, f_n.

As Eq. 1a is theoretically derived (as will be seen subsequently), any consistent set of units may be used. For reasons that will be useful subsequently, Eq. 1a can be rewritten as

$$T_o = 2\pi\left\{\frac{\sigma'_{v_o} H}{E_{t_i} g}\left[4\left(\frac{H}{B}\right)^2 + \left(\frac{12+12\nu}{5}\right)\right]\right\}^{0.5} . \quad \textbf{(1b)}$$

Investigation of the Original T_o Equation

Origin

The original source of Eq. 1a was original research presented in Horvath (1995). It was based on the assumption that an embankment consisting primarily of EPS-block geofoam can be modeled as a single-degree-of-freedom (SDOF) system (see page 140 in Horvath (1995) for model illustrations). The fundamental period of any SDOF system is given by the following equation:

$$T_o = 2\pi\left[m\left(\frac{1}{k}\right)\right]^{0.5} \quad \textbf{(2a)}$$

where:

k = spring stiffness of the SDOF system,
m = mass of the SDOF system,

and the remaining terms have been defined previously.

It is traditional to work with the mass, m, of a SDOF system and, in fact, to use mass in any problem involving dynamic loading or behavior. However, for reasons that will become clear subsequently it is better to work in terms of weight instead of mass for this particular application. Therefore Eq. 2a becomes:

$$T_o = 2\pi\left[\left(\frac{W}{g}\right)\left(\frac{1}{k}\right)\right]^{0.5} \quad \textbf{(2b)}$$

where W = weight of the SDOF system. The issue, then, is how to define the parameters in Eq. 2b for an EPS-block geofoam embankment.

Derivation

In deriving Eq. 1a from Eq. 2b, Horvath (1995) assumed that the spring stiffness, k, in Eq. 2b was best modeled using the theoretical equation for the maximum transverse displacement (deflection), Δ, of a fixed-end cantilever beam subjected to a transverse concentrated force, P, at its free end. Spring stiffness, k, in the context of beam displacement as used here is defined conceptually by the following equation:

$$k = \frac{\text{applied force, } P}{\text{displacement, } \Delta \text{, as a result of } P} \quad \textbf{(3)}$$

Both flexural and shear contributions to displacement were assumed. This is sometimes referred to as a 'Timoshenko' beam as opposed to an 'Euler' beam that is used for 'simple-beam' theory in which the shear contribution to beam displacements is neglected (Horvath 2002). Therefore, in the more-general Timoshenko-beam case k has two components:

- k_F which is the contribution to spring stiffness due to flexure (bending) effects and
- k_S which is the contribution to spring stiffness due to shear effects.

Because these two contributions are additive, i.e. they both occur as a result of the same force P, these two springs, k_F and k_S, can be visualized as acting in series. As is well known, the equivalent spring stiffness (which, in this case, is defined simply as k) of two springs in series is:

$$k = \frac{1}{\frac{1}{k_F} + \frac{1}{k_S}} . \tag{4}$$

Thus $1/k$ as used in Eq. 2b is

$$\frac{1}{k} = \frac{1}{\frac{1}{\frac{1}{k_F} + \frac{1}{k_S}}} = \frac{1}{k_F} + \frac{1}{k_S} \tag{5}$$

and Eq. 2b itself can be rewritten as

$$T_o = 2\pi \left[\left(\frac{W}{g} \right) \left(\frac{1}{k_F} + \frac{1}{k_S} \right) \right]^{0.5} . \tag{6}$$

Thus what remains is to evaluate k_F and k_S using the conceptual definition stated in Eq. 3.

Considering first the flexure component, k_F, the maximum flexural transverse displacement, Δ_F, of a cantilever beam with a concentrated transverse load P at its free end is

$$\Delta_F = \frac{PL^3}{3EI} \tag{7}$$

where:

E = Young's modulus of the beam material,
I = moment of inertia of the beam,
L = beam length,

and the remaining terms were defined previously. In the case of an embankment incorporating EPS geofoam, $E = E_{t_i}$; $L = H$; and $I = B^3/12$ (note that I is per unit length of the embankment). Thus Eq. 7 becomes

$$\Delta_F = \frac{4PH^3}{E_{t_i} B^3} . \tag{8}$$

Combining Eqs. 3 and 8 and simplifying produces the desired spring stiffness of the flexural component:

$$k_F = \frac{P}{\Delta_F} = \frac{PE_{t_i}B^3}{4PH^3} = \frac{E_{ti}B^3}{4H^3}\ . \qquad \textbf{(9)}$$

Finally, the desired final result for use in Eq. 6 is:

$$\frac{1}{k_F} = \frac{4H^3}{E_{t_i}B^3}\ . \qquad \textbf{(10)}$$

Going through a similar process for the shear-spring component:

$$\Delta_S = \left(\frac{6}{5}\right)\frac{PL}{GA} \qquad \textbf{(11)}$$

where:

A = beam cross-sectional area,
G = shear modulus of the beam material,

and the remaining terms have been defined previously. In this application, *A* = *B* and *L* = *H* so Eq. 11 becomes

$$\Delta_S = \left(\frac{6}{5}\right)\frac{PH}{GB}\ . \qquad \textbf{(12)}$$

Combining Eqs. 3 and 12 and simplifying produces the desired spring stiffness of the shear component:

$$k_S = \frac{P}{\Delta_S} = \left(\frac{5}{6}\right)\frac{PGB}{PH} = \left(\frac{5}{6}\right)\frac{GB}{H}\ . \qquad \textbf{(13)}$$

Finally:

$$\frac{1}{k_S} = \left(\frac{6}{5}\right)\frac{H}{GB}\ . \qquad \textbf{(14)}$$

Although Eq. 14 could be used in Eq. 6 it is possible to simplify things further by eliminating the shear modulus, *G*, as an explicit variable in this problem. Recognizing that in the current application of interest the beam represents the entire EPS mass means that the beam material is the EPS. For a linear-elastic material as the EPS is assumed to be in this application:

$$G = \frac{E}{2(1+\nu)} = \frac{E_{t_i}}{2(1+\nu)}\ . \qquad \textbf{(15)}$$

Combining Eqs. 14 and 15 yields the required input for Eq. 6:

$$\frac{1}{k_S}=\left(\frac{6}{5}\right)\frac{2H(1+\nu)}{E_{t_i}B}=\left(\frac{12}{5}\right)\frac{H(1+\nu)}{E_{t_i}B}\,. \tag{16}$$

Finally, combining Eqs. 6, 10 and 16, and rearranging terms yields:

$$T_o=2\pi\left[\left(\frac{W}{g}\right)\left(\frac{4H^3}{E_{t_i}B^3}+\left(\frac{12}{5}\right)\frac{H(1+\nu)}{E_{t_i}B}\right)\right]^{0.5}=2\pi\left\{\left[\frac{WH}{E_{t_i}gB}\right]\left[4\left(\frac{H}{B}\right)^2+\left(\frac{12}{5}\right)(1+\nu)\right]\right\}^{0.5}\,. \tag{17}$$

One last step is recognizing that *W*/*B* per unit length of the embankment = σ'_{v_o} so Eq. 17 can be rewritten in its final form:

$$T_o=2\pi\left\{\frac{\sigma'_{v_o}H}{E_{t_i}g}\left[4\left(\frac{H}{B}\right)^2+\left(\frac{12}{5}\right)(1+\nu)\right]\right\}^{0.5} \tag{18}$$

which is the same as Eq. 1a and thus completes the desired derivation.

However, before closing this discussion it is of interest to note that there are two alternatives to Eqs. 1a/18 that should be discussed for the sake of completeness. Both of these alternatives were raised and discussed during development of the CA/T Contract C09C2 Project Design Criteria.

Alternative Equations for T_o

Rigorous Solution

The first alternative is based on the fact that Eq. 11 used to derive the shear contribution to the SDOF system spring stiffness is actually an approximate one. The approximation derives from the use of the 6/5 factor. In reality, this factor is not a constant. The theoretically rigorous version[g] of Eq. 11 is

$$\Delta_S=\left[\frac{12+11\nu}{10(1+\nu)}\right]\frac{PL}{GA} \tag{19}$$

where all terms have been defined previously. Thus as Poisson's ratio, ν, varies from 0 to 0.5 the factor in brackets in Eq. 19 varies from 1.20 to 1.17. Clearly, however, 6/5 = 1.20 is a reasonable constant-value approximation to this variable quantity. The maximum possible error is approximately 3%. Note, however, that because Poisson's ratio for EPS in its elastic range is much closer to 0 than 0.5 the actual error for EPS-block-geofoam embankments is even less than 3% (as will be seen subsequently). Regardless of the value of Poisson's ratio, because Eq. 11 is both reasonably accurate and simpler to use than Eq. 19 it is apparent why Eq. 11 is typically what is seen and used in practice.

Nevertheless, if Eq. 19 and not Eq. 11 is used to develop the SDOF model considered in this technical note then the governing equation becomes (details omitted):

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[g] See, for example, page 207 of Timoshenko and Gere (1972).

$$T_o = 2\pi\left\{\frac{\sigma'_{v_o} H}{E_{t_i} g}\left[4\left(\frac{H}{B}\right)^2 + \left(\frac{12+11\nu}{5}\right)\right]\right\}^{0.5} \tag{20}$$

which is better compared to Eq. 1b (the slightly rewritten version of Eq. 1a). The slight difference in the shear-related terms is quite clear.

Japanese Design Equation

The other alternative noted here is the equation that appears in various Japanese publications dealing with EPS-block-geofoam embankments. Using notation consistent with that used throughout this report, the Japanese equation for T_o can be expressed as

$$T_o = 2\pi\left\{\frac{\sigma'_{v_o} H}{E_{t_i} g}\left[4\left(\frac{H}{B}\right)^2 + 1 + \left(\frac{12}{5}\right)(1+\nu)\right]\right\}^{0.5} \tag{21a}$$

or, to facilitate comparison with Eqs. 1b and 20:

$$T_o = 2\pi\left\{\frac{\sigma'_{v_o} H}{E_{t_i} g}\left[4\left(\frac{H}{B}\right)^2 + \left(\frac{12+12\nu}{5}\right) + 1\right]\right\}^{0.5} . \tag{21b}$$

The key difference between Eq. 21b and Eqs. 1b (the original, slightly-approximate model) and 20 (the more-rigorous version of original model) is the appearance of the number '1' in Eq. 21b within the bracketed expression that incorporates the spring stiffnesses.

Considering how common terms were factored out during the derivation of the final equation for T_o in the original model (see Eq. 17), the additional term appearing in the Japanese version is not actually '1' but

$$\frac{H}{E_{t_i} B} . \tag{22}$$

Working backward further and assuming the term in Eq. 22 represents some kind of additional spring stiffness, k^*:

$$\frac{1}{k^*} = \frac{H}{E_{t_i} B} \tag{23}$$

which means that the transverse displacement, Δ^*, associated with additional spring stiffness is

$$\Delta^* = \frac{P}{k^*} = \frac{PH}{E_{t_i} B} . \tag{24}$$

It is not obvious what the additional term reflected in Eq. 22 is intended to represent as there are no other theoretical contributions to beam stiffness per se other than flexure and shear. From Eq. 24 it appears to be a contribution to beam displacement that is somehow related to shear (compare the similar form and content of Eqs. 23 and 16). Because Eq. 21a (and, by implication, it's more-insightful equivalent, Eq. 21b) has appeared in numerous Japanese publications over a period of more than 10 years the possibility that there is a typographical error in the equation appears to be small[h]. It is also possible that that this additional, presumably shear-related stiffness is some empirical adjustment based on extensive Japanese research which includes numerical analyses and reduced- and full-scale shake-table tests. However, at this time no definitive explanation is known so Eq. 21a and its equivalent, Eq. 21b, are simply provided as information without further conjecture, explanation or comment.

Comparison of Alternative Equations

It is not the purpose of this technical note to perform a detailed comparison of the current model (Eqs. 1a/1b) and its two alternatives (Eqs. 20 and 21a/21b). Nevertheless, it is of interest to explore the sensitivity of the results calculated using the three different equations for a typical design case that arose on CA/T Contract C09C2. The specific parameter values are omitted here but the overall problem geometry can be seen in figures contained in Riad et al. (2003b) and Horvath (2003). The following were the calculated results:

- Equation (Eqs. 1a/1b) for the original, slightly-approximate model as developed originally by Horvath and first published in Horvath (1995), and as used in both the NCHRP research (Stark et al. 2000, 2002) and for CA/T Contract C09C2: T_o = 1.17 seconds.

- Alternative equation No. 1 (Eq. 20) which was also developed originally by Horvath (2001, unpublished) and is a more-rigorous version of the preceding equation: T_o = 1.17 seconds (no change in significant figures).

- Alternative equation No. 2 (Eqs. 21a/21b) as used in Japan: T_o = 1.24 seconds (6% increase over preceding two equations).

For CA/T Contract C09C2, the American Association of State Highway and Transportation Officials (AASHTO) bridge-design code was used for all aspects of seismic design of the EPS-block-geofoam embankments. This code uses empirical formulas, not site-specific seismic response spectra, to relate the T_o of a structure to its seismic amplification. Because of the way in which T_o influences the calculation of the seismic force in the AASHTO formulas, the Japanese equation (Alternative No. 2) produced a seismic force in this example that is approximately 5% smaller in magnitude than the equation used for CA/T Contract C09C2. The final results are, therefore, close in magnitude with the CA/T project methodology yielding a slightly more-conservative result.

Concluding Comments

The results from the single example problem considered here must not be considered a general conclusion. Nevertheless, considering the soundness of the theoretical derivation of the equation given originally in Horvath (1995) and later in the NCHRP Project No. 24-11 reports (Stark et al. 2000, 2002) and as adopted by the CA/T project, it appears that use of this equation (Eq. 1a) is appropriate considering the current state of knowledge regarding the behavior of EPS-block-geofoam embankments under seismic loading. Alternatively, a somewhat more-rigorous version

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[h] Small but not zero. It is relevant to note that virtually all research into EPS-block geofoam in Japan, especially that related to seismic behavior, passes through one organization, the EPS Development Organization (EDO). Thus the possibility of some systematic, institutional error is possible.

of this equation (Eq. 20) could be used. As always, these suggestions are contingent upon future research which may warrant changes in equations and analytical methodologies.

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