Manhattan College

Center for Geotechnology

Soil-Structure Interaction Research Project

Analysis of
Vertically Anchored Foundation Elements

Report No. CGT-2001-3

by

John S. Horvath, Ph.D., P.E.
Professor of Civil Engineering
Director/Center for Geotechnology

Manhattan College
School of Engineering
Center for Geotechnology
Bronx, New York
U.S.A.

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Prof. John S. Horvath, Ph.D., P.E.
Manhattan College
Civil Engineering Department
Bronx, New York 10471-4098
U.S.A.

email: <jhorvath@manhattan.edu>
personal webpage: <www.manhattan.edu/~jhorvath/>
direct-dial telephone: +1-718-8627177
telefax: +1-718-8628035

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The Manhattan College School of Engineering *Center for Geotechnology* and Its Mission

The Manhattan College School of Engineering *Center for Geotechnology* (CGT) is a unique organization that strives to be more than the typical academic research center or institute. It was founded in 2001 at the initiative of Prof. John S. Horvath, Ph.D., P.E. of the Civil Engineering Department who serves as its first Director. The CGT is the result of Prof. Horvath's evolutionary realization after almost 30 years of geotechnical engineering practice that the explosive growth in geotechnical and geoenvironmental engineering technology has made it difficult for the engineering practitioner to keep abreast of new technical developments. The traditional academic approach of simply publishing research results in narrowly disseminated technical reports and papers (a philosophy of "if you print it, they will learn") has proven to be an increasingly ineffective way of reaching practitioners and moving the state of art to the state of practice. The critical need for a total rethinking of how life-long continuing education is achieved not only for engineering practitioners but academicians themselves is evidenced by the appearance of "teach-the-teacher" training courses in drilled shaft foundations and geosynthetics beginning in the late 1980s. If even academicians cannot keep up with new developments by reading journal papers and conference proceedings, how can practitioners be expected to? The stagnation of geotechnology also affects current engineering students and perpetuates the cycle. The desirability of involving the practitioner in the process of formulating research programs so that they may have a more direct and immediate benefit to practice is also something that is now recognized more and more.

The CGT seeks to address the current need for effective, meaningful continuing education by recognizing that the cycle of growth for any technology has three interdependent components, what can be called the "trilogy of technology". Like a three-legged stool, each of these components must be of equal length and strength if a given technology is to succeed. Thus the CGT has adopted a holistic strategy of supporting geotechnology growth by recognizing the need to concurrently address:

- *Technology advancement* through research and development that involves not only the engineering practitioner but also other end users of geotechnology such as construction contractors and material manufacturers to the greatest extent practicable.

- *Technology transfer* through education of engineers, contractors and manufacturers in a multi-faceted, proactive way.

- *Technology documentation* through standards development so that all end users (practitioners, contractors and manufacturers) of a given technology work to a common set of guidelines.

This trilogy of technology growth is the cornerstone of all activities of the CGT. It is important to note that the interaction of these three components, which is embodied in the CGT logo that is shown on the cover of this report, is never completed but assumes a constant cycle that leads to continuous growth of a technology.

The CGT receives no direct financial support from Manhattan College for any of its activities. Thus the success and growth of the CGT is totally a function of outside funding from individuals and organizations whose philanthropic philosophies are consistent with the stated goal of the CGT to treat technology growth in a more holistic fashion than is typically done in academia and considers the entire process from research to standards with end-user input at all stages. In addition, as part of its mission to promote technology transfer through education to the greatest extent practicable the CGT is willing to partner with industry and other academic institutions not only in research but also technology transfer and standards activities on any topic relevant to geotechnical or geoenvironmental engineering. The new Manhattan College School of Engineering William J. Scala Academy Room, which is located on the main floor of the Leo Engineering Building and available for CGT activities, offers modern facilities for hosting technology transfer activities. One benefit of Manhattan College's location on the northern edge of New York City is that it is quite accessible (including free off-street parking adjacent to Leo Engineering Building) from both within and outside the City. More information about the CGT can be found on the Internet at <www.engineering.manhattan.edu/civil/CGT.html>.

Preface

The origins for this report go back to the early 1980s when I was involved in a number of projects in engineering practice where uplift loading on various types of foundation elements (spread footings, slabs on grade) was an important design consideration. The use of vertically oriented ground anchors was a common design solution to deal with potential uplift on such foundation elements. Over time, I became troubled by the ambiguities, inconsistencies and contradictions in the conventional design approach for such anchors that was used in routine practice at that time.

My move to academia on a full-time basis in 1987 allowed me the time to research the topic of vertically anchored foundation elements in some detail. This was an early research priority of mine. My research efforts were both self-motivated and self-funded (ground-anchor suppliers such as Dywidag curiously showed no interest in my work) so there was no externally imposed agenda that was skewed toward one particular product or outcome. My only self-imposed goal was to better understand the true mechanics of vertically anchored foundation elements with an eye toward developing a more rational design procedure. The lack of funding did, however, significantly constrict what could be done, e.g. physical testing of full-scale anchors, which would have been highly desirable, was out of the question.

I published the results of my research on this topic in 1990 as a contribution to the Manhattan College Civil Engineering Department geotechnical engineering program (CE/GE) research report series that I had established in 1987. This particular report, which was issued only in traditional hardcopy (paper) format, is no longer available. This report series itself was discontinued at the end of 2000 in anticipation of the formation of the Manhattan College *Center for Geotechnology* (CGT) and the new CGT research report series.

The basic elements of my 1990 CE/GE report were summarized, edited, updated and published in a preprint paper that was prepared by me for a technical session at the 72nd Annual Meeting of the Transportation Research Board (TRB) that was held in January 1993 in Washington, D.C. This preprint paper also formed the basis of my oral presentation at a technical session. Unfortunately, TRB did not deem the paper worthy of subsequent formal publication in their *Transportation Research Record* series so dissemination of the contents of my paper was quite limited (this was back in the days when TRB made available to the public only about 100 paper copies of each preprint).

Since 1993, I have not seen the topic of vertically anchored foundation elements addressed to any significant extent in the literature even though it remains a vitally important topic in practice. Thus I felt that it would be a worthwhile to make the contents of my TRB paper more readily available via the Internet by issuing a report in the new CGT research report series. Although the contents of the original TRB preprint paper have been substantially edited and updated to hopefully improve and clarify the presentation, the basic results and conclusions presented herein are the same as those in my original 1990 report and as presented at the TRB annual meeting in 1993. I feel that my opinions on this subject have stood the test of more than a decade of time pretty well.

John S. Horvath, Ph.D., P.E.
Bronx, New York, U.S.A.
June 2001

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Executive Summary

Vertically oriented steel ground anchors are frequently used to restrain various types of *foundation elements* (spread footings, mats (rafts), slabs on grade, etc.) from uplift forces. This is referred to as an *anchored foundation*. The anchors are usually prestressed after installation (*prestressed anchored foundation*) as opposed to leaving them unstressed (*passive anchored foundation*) so that foundation-subgrade separation does not occur when post-construction uplift forces are applied to the anchored foundation.

Prestressing produces tensile stress in the anchors and compressive stress between the foundation element and subgrade. For simplicity, the conventional design method used traditionally for prestressed anchored foundations assumes that the problem is statically determinate and can thus be solved only by considering static-force equilibrium. Deformations and soil-structure interaction of what is actually a statically indeterminate problem are neglected. The conventional design method further assumes that no net stress change occurs in the anchors as a post-construction uplift force is applied to a prestressed anchored foundation. Rather, the foundation-subgrade contact stress is assumed to decrease in direct and linear proportion to the increase in applied force so that the net change in anchor stress is zero.

The premise of this report is that the static indeterminacy and concomitant soil-structure interaction aspects of the actual problem cannot be neglected. Consequently, the simple analytical assumptions traditionally made for prestressed anchored foundations are incorrect because stiffnesses and deformations of the subgrade and anchors are neglected. As a result, an uplift force will always cause some additional tensile stress in a prestressed anchor. Whether or not this stress increase is relatively significant in magnitude and thus important to consider turns out to depend on the relative stiffnesses of anchors and subgrade.

The results presented in this report, which are based on numerical analyses, indicate that this relative increase in anchor stress increases with decreasing subgrade stiffness, with a secondary dependency on anchor stiffness. The anchor stress increase was found to be effectively zero (and thus insignificant in practical applications) for a simulated hard, sound rock subgrade but relatively large (of the order of 50% and thus potentially significant in practical applications) for a sand subgrade. Therefore, the conclusion reached in this report is that prestressed anchored foundations should always be analyzed as a statically indeterminate soil-structure interaction problem. A relatively simple physical and mathematical model based on using springs to represent the anchor and subgrade stiffnesses is proposed as a simple method for analyzing the behavior of anchored foundations in routine practice. This model can also be used when initially unstressed (passive) ground anchors are used.

Also considered in this report as a secondary topic is the response of prestressed anchored foundations to downward loading. This problem has not received much attention in the past but could be useful in some practical applications to either preload a new foundation element to reduce subsequent settlement under downward superstructure loads or to correct the tilt of an existing foundation. The results of numerical analyses presented in this report suggest that the practical benefit of using prestressed ground anchors to preload foundation elements for downward forces increases with decreasing subgrade stiffness which is precisely the conditions for which preloading is most often considered. The simple spring model proposed in this report for uplift forces can also be used for downward forces.

Finally, the results presented in this report produced an unexpected outcome that is of interest for prestressed anchored foundations that are subjected to both uplift and downward forces after construction. This typically occurs when spread footings bearing on bedrock are used to support relatively slender high-rise buildings. It appears that using prestressed anchors to resist potential uplift forces (typically from low-probability extreme wind and seismic loads) may inadvertently lead to a condition of subgrade overstress when the foundation element is subjected only to downward forces

(which are typically from dead and live loads within the structure and thus exist almost constantly). This issue does not appear to have been addressed in the literature. The simple spring model proposed in this report can be used to analyze this problem as well.

INTRODUCTION

Background

Steel ground anchors (typically either deformed bar or wire strand) with vertical or near-vertical orientation have been used for many years to resist uplift forces applied to various types of structural elements (*foundation elements*) supported on or in the ground. Examples include footings for building columns and electrical transmission towers, mat (raft) foundations, slabs on grade, base slabs of "box" tunnels constructed using the cut-and-cover method and hydraulic structures (dams and spillways). For simplicity, in this report such anchored structural elements are referred to collectively and generically as *anchored foundation elements* or simply *anchored foundations*. The uplift forces applied to anchored foundations may be relatively constant (e.g. uplift water pressures beneath a hydraulic structure) but more typically are not and result from some extreme natural event involving an earthquake, water or wind with a relatively low probability of occurrence during the design life of the foundation element.

Regardless of the specific type of anchored foundation, a common feature is that each anchor is usually prestressed after installation by jacking against the foundation element and *locking off* (connecting to the foundation element) with the prestress intact to create a *prestressed anchored foundation*. Upon initial consideration, prestressing would appear to be illogical in this application as it imparts an initial tensile stress in the anchor which will then be subjected to further, theoretically additive tensile stresses due to design uplift forces. The justification typically given for prestressing, as opposed to using a non-prestressed (passive) anchor (*passive anchored foundation*), is the desire to prevent vertical separation between the foundation element and underlying ground (subgrade) as an uplift force is applied.

Despite the widespread successful use of prestressed anchored foundations, the author's practical experience with them during the 1980s raised a number of questions and concerns about the analytical assumptions and methods typically used in routine practice, especially the prestressing issue. This led to the research project that was originally presented in Horvath (1990), synopsized and updated in Horvath (1993), and is now further edited and updated in this report.

Terminology

The term *design load* as used in this report is defined as the maximum unfactored (service) load applied to the foundation element under some load case. Design loads may consist of both forces and couples (moments), but forces tend to predominate in the types of problems considered in this report. Although the primary emphasis of this report is on uplift forces, downward forces as well as combined uplift and downward forces will also be considered for reasons discussed subsequently.

Using the traditional allowable stress design (ASD, also known as working stress design (WSD)) method, anchors are usually designed to have some maximum allowable axial force or stress under the design load[a]. In this report, it is assumed that anchor failure would only be structural, i.e. by yield of the anchor steel. Geotechnical failure by anchor pullout within its bond length is assumed not to occur. Therefore, the term *yield* as used in this report applies only to the anchor steel and the anchor design load equals the theoretical axial force required to cause structural yield of the anchor steel divided by some safety factor. However, the concepts discussed in this report are fully valid if anchor failure is defined by pullout and not structural yield.

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[a] Because of their relative slenderness, ground anchors are generally assumed to behave structurally as "truss members", i.e. be subjected to axial forces only and no bending moments. A notable exception in recent years has been when anchors have been used for soil nailing applications.

The *lock-off load* is the tensile force in an anchor at the end of the prestressing and prooftesting process, and is expressed as a percentage of the design load. The author's specific practical experience during the 1980s with prestressed anchored spread footings supporting high-rise buildings in New York City has been to lock off the anchor prestress at 100% of the design load which was usually taken to be one-half of yield of the anchor steel, i.e. a safety factor of two against structural failure of the anchor.

CONVENTIONAL DESIGN METHOD

Basic Concept

In traditional design practice, analysis of prestressed anchored foundations is treated as a simple statically determinate problem involving force equilibrium only. The free-body diagram of a typical prestressed anchored foundation is shown in Figure 1. P_F is the externally applied uplift design load (force) to be resisted, P_A is the resultant force exerted on the foundation element by all anchors and P_S is the resultant contact (bearing) force exerted on the foundation element by the ground (subgrade). Note that immediately after anchor prestressing, prooftesting and lock off $P_F = 0$ thus $P_S = P_A$. Also defined in this figure is w, the vertical displacement of the foundation element relative to its initial position <u>after</u> anchor prestressing and lock off but before application of any uplift load P_F. The positive sense of each variable is as shown in the figure.

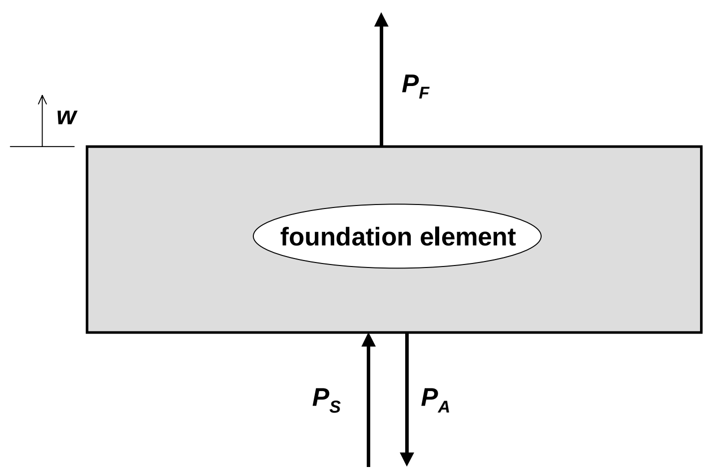


Figure 1. Free-Body Diagram: Prestressed Anchored Foundation

As noted previously, prestressing creates an initial tensile stress in each anchor and an uplift force also tends to produce a tensile stress in each anchor. Thus, in principle, the signs of the stresses are additive. Theoretically then, a prestressed anchor locked off at 100% of a design load of one-half of yield could actually yield when the full design uplift force is applied. This is the paradox that concerned the author during the 1980s.

The hypothesis offered traditionally to resolve this apparent dilemma is based on applying simple vertical force equilibrium to the force system shown in Figure 1:

$$\sum^{\uparrow+} \text{vertical forces} = P_F + P_S - P_A = 0 \qquad \textbf{(1)}$$

and assuming that two physical phenomena occur simultaneously as an uplift force, P_F, is applied to the foundation element:

- The increase in anchor tensile force, P_A, caused by P_F causes the anchors to elongate and the foundation element to displace upward (+*w*).

- Upward displacement, +*w*, reduces the contact force, P_S, between the foundation element and subgrade that was induced by prestressing (recall $P_S = P_A$ initially). This in turn reduces the prestress portion of the anchor force, P_A, which causes the anchors to shorten and the foundation element to displace downward (-*w*).

The crucial further assumption is that these two phenomena are not only concurrent but cancel each other exactly, i.e. an increase in P_F is matched exactly by a decrease in P_S, so that P_A remains unchanged. This also implies that there is no net vertical displacement of the foundation element, i.e. $w = 0$, so the foundation element would never move vertically and therefore certainly never lift off the underlying subgrade which is desirable.

As an observation, note that according to the above hypothesis when the full design uplift force, $P_{F(max)}$, is reached the subgrade contact force, P_S, would just be zero. Thus any further application of uplift force beyond $P_{F(max)}$ would simultaneously cause a net upward movement of the foundation (+*w*) with concomitant loss of foundation element-subgrade contact and an increase in anchor force, P_A.

Shortcomings

In the author's opinion, this traditional hypothesized behavior and the resulting conventional design method that derives from it is difficult to accept. Engineering intuition suggests that it is unlikely that an increase in uplift force, P_F, would be matched exactly by a decrease in subgrade-foundation contact force, P_S, for every combination of foundation element, subgrade and anchor conceivable in practice. This hypothesis is too simplistic because it neglects the soil-structure interaction aspects of the problem, namely, absolute and relative stiffnesses (force-displacement relationships) of both subgrade and anchors. In general, the vertical displacement, *w*, resulting from a unit force applied to the anchors, P_A, versus subgrade, P_S, will be different. As a result, application of an uplift force, P_F, is not canceled out exactly by a reduction in foundation element-subgrade contact force, P_S. Consequently, the tensile force, P_A, in the anchors caused by an uplift force, P_F, will always be additive to some extent to that pre-existing from prestressing. As a result, there will always be some net increase in anchor force, P_A, above lock off. It follows that the anchors will also have a net elongation (as well as some possible movement within their bond length) and the foundation element a net upward displacement, +*w*. Key questions are:

- How large is the error (which is always on the unconservative side) between reality and the assumptions of the conventional design method, and how does the error vary with problem variables?

- Does this error have a significant practical impact on current design methods or should new methods be developed?

- Will lift off of the foundation element from the subgrade occur?

These issues have great practical importance. All engineering analyses, especially those in geotechnical engineering, are approximate of necessity and thus "wrong" to some degree. What is important is whether or not the approximations built into engineering practice result in significant deviations from reality and whether these deviations are conservative or unconservative.

The conclusion reached by the author (Horvath 1990) regarding anchored foundations is that an analytical approach that explicitly considers the soil-structure interaction aspects of the problem should always be used in practice. The key information on which this conclusion is based is summarized in this report. Theoretically, existing numerical methods consisting of a finite-element or finite-difference solution of a geotechnical continuum model could be used. However, such methods, although much easier to use than in the past, are still much too complex and time consuming for routine practice. This means that there is a need for an analytical model that strikes a better balance between accuracy and ease of use than existing alternatives. The remainder of this report is devoted to development and verification of just such a model.

PROPOSED NEW ANALYTICAL MODEL

Concept and Development

A new physical and mathematical model is proposed for analysis of prestressed anchored foundations that includes explicit consideration of anchor and subgrade stiffnesses. This model is relatively simple and suitable for manual calculation, yet captures the essential elements of the problem.

The basic concept of this model is that the aggregate force-displacement behavior of the anchors and subgrade can each be represented by a single spring. Referring to Figure 1, P_S and P_A are now viewed as spring forces that are a function of foundation displacement, w, and each with an initial magnitude P^* after anchor prestressing, prooftesting and lock off. Upon application of the uplift force, P_F, these spring forces are assumed to vary in accordance with:

$$P_A = P^* + (k_A \cdot w) \quad \textbf{(2a)}$$

$$P_S = P^* - (k_S \cdot w) \quad \textbf{(2b)}$$

where k_A and k_S are the equivalent spring stiffnesses of the anchors and subgrade respectively. These stiffnesses have dimensions of force per length and are simply the slopes of the respective force-displacement curves. In this report the springs are assumed to be linear although this is not necessary and, in fact, is unlikely for the subgrade spring.

In practice, it is necessary to put limits on both P_A and P_S. The following is suggested for the anchors:

$$0 \le P_A \le P_{A_u} \quad \textbf{(3)}$$

where P_{A_u} is the force required to cause anchor failure. Note that although it has been assumed in this report that anchor failure is caused by steel yield as opposed to bond failure, this does not have to be the case in general. P_{A_u} could be taken to be the pullout force of the anchor if it is more conservative than steel yield in a given application.

For the subgrade:

$$0 \le P_S \le P_{S_u} \quad \textbf{(4)}$$

where P_{S_u} is the force required to cause a bearing failure of the subgrade. This can be calculated using conventional methods given in geotechnical textbooks. Information in Horvath (2000a, 2000c) is also useful for estimating P_{S_u} .

Assuming that neither the anchors nor subgrade fails, equations 1, 2a and 2b can be combined to produce the following:

$$P_F = w \cdot (k_A + k_S) \quad \textbf{(5)}$$

which, after rearranging, becomes:

$$w = \frac{P_F}{(k_A + k_S)} \quad \textbf{(6)}$$

Use in Practice

Equations 5 and 6 can be used in practice to estimate, respectively, the total applied uplift force, P_F, required to cause a given magnitude of upward foundation displacement, w, and the upward foundation displacement, w, that will occur as a result of a given magnitude of applied uplift force, P_F, provided that equations 3 and 4 are satisfied for the value of w used or calculated. In addition, the magnitude of upward foundation displacement required to just cause the anchors to fail can be obtained from Equation 2a by setting $P_A = P_{A_u}$. The magnitude of upward foundation displacement required to just cause the foundation to separate from the subgrade can be obtained from Equation 2b by setting $P_S = 0$. Equations 5 and 6 also provide insight into the relative apportioning of load between subgrade and anchors by virtue of the relative magnitudes of spring stiffnesses k_A and k_S. The magnitudes of forces carried by subgrade and anchors can be calculated by substituting a given magnitude of vertical foundation displacement in equations 2a and 2b.

The question of how to evaluate the equivalent anchor and subgrade spring stiffnesses k_A and k_S in practice is an important one. As a first approximation, one can simply sum AE/L over all the anchors for a given foundation element where A = anchor cross-sectional area, E = Young's modulus of anchor steel and L = the free (unbonded) length of each anchor. This neglects the reduction in effective anchor stiffness that would be caused by any anchor-ground displacement (which may be time dependent) within the bond length. This issue should be studied as part of future research and this simple suggestion for calculating anchor stiffness may require modification based on the findings of that research.

With regard to the subgrade stiffness, note that it is actually the stiffness in unloading, not loading. However, it can be estimated by performing a settlement analysis using any number of established techniques depending on the type of subgrade material but making sure that an appropriate unloading modulus is used as opposed to a loading modulus. The equivalent spring stiffness is simply the slope of the load-settlement curve generated.

NEW-MODEL VERIFICATION: BACKGROUND

Scope of Study

Case-history data to verify the proposed spring model was unavailable or at least unknown to the author at the time the original research was performed in the late 1980s. Likewise, there was no funding available for full-scale installation and load testing which would be the most desirable way to verify the model. Consequently, additional study of limited scope was performed by the author to gain preliminary insight into the problem and provide results against which the proposed spring model could be compared. This additional work was limited to numerical analysis of a geotechnical continuum using the finite-element method. The complete results were published in Horvath (1990).

A limited number of variables were considered in this numerical study:

- relatively stiff (sound, crystalline bedrock) versus moderately compressible (medium-dense to dense sand) subgrades,
- 150 kip/in^2 (1000 MPa) versus 60 kip/in^2 (400 MPa) yield strength, f_y, of the anchor steel which is typical of the widely used Dywidag deformed bars,
- prestressed versus passive anchors and
- uplift versus downward loading.

This last variable of downward loading was included to investigate the little-explored concept of using prestressed anchors to preload a foundation element. Also, many foundation elements are designed for both uplift and downward load cases (this is typical of high-rise buildings, for example), so it is of interest to see how an anchored foundation behaves in compression.

The simulated foundation element in all cases discussed in this report was a footing. Although not exhaustive, the cases studied were judged to span a reasonable range in conditions, i.e. combinations of subgrade and anchor stiffnesses, that might occur in practice.

Software Used

The finite-element analyses were performed using a program named *SSTIPNH*™, the author's personal and personally modified microcomputer version of the mainframe program *SSTIPN* (*User's* 1979). Both *SSTIPN* and *SSTIPNH* analyze geotechnical continua under plane-strain conditions. It is both relevant and important to note that the anchor prestressing simulations that were crucial to this study were only possible using source-code changes that were incorporated into *SSTIPNH* by the author. Such changes were possible because at the time (late 1980s) that Manhattan College purchased the *SSTIPN* code from The Virginia Polytechnic Institute and State University (Virginia Tech), the original FORTRAN source code was provided.

Problem Geometry

Because *SSTIPNH* can only analyze planar problems, a vertical slice of unit width (one foot in this case) through the simulated footings was modeled. This effectively implied that a continuous (strip) footing was being modeled. The overall problem geometry is shown in Figure 2 for both the soil and rock footings. The footings were 2 feet (610 mm) and 3 feet (900 mm) thick respectively. All analyses were performed using imperial units. Approximate SI equivalents are given in the figure.

As shown in this figure, for the analyses involving a soil subgrade (left side of Figure 2) the anchors were assumed fixed at a rigid base 50 feet (15 m) below the bottom of the footing. This meant that there was a 52 foot (16 m) total unbonded length (the top of each anchor was assumed fixed after prestressing and lock-off at the top of the footing). The bonded length was not modeled explicitly.

For the analyses involving a rock subgrade (right side of Figure 2), a 10 foot (3 m) unbonded length below foundation level was assumed and a 20 foot (6 m) bond length was modeled explicitly. Perfect adhesion was assumed between anchor and rock within the bonded length. In this case, the bottom of the anchor was 20 feet (6 m) above the assumed rigid base.

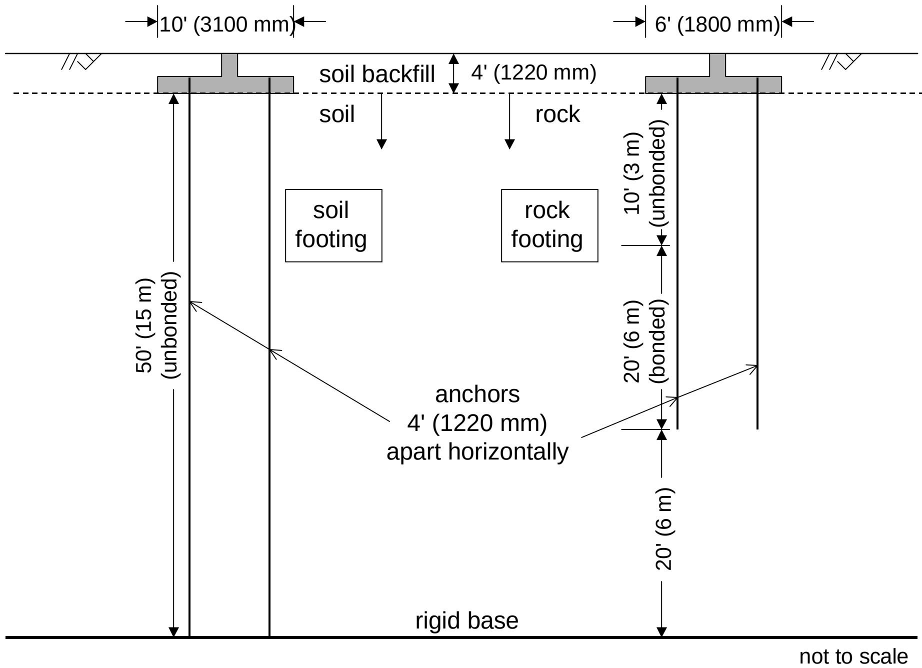


Figure 2. Problem Geometry for Finite-Element Analyses

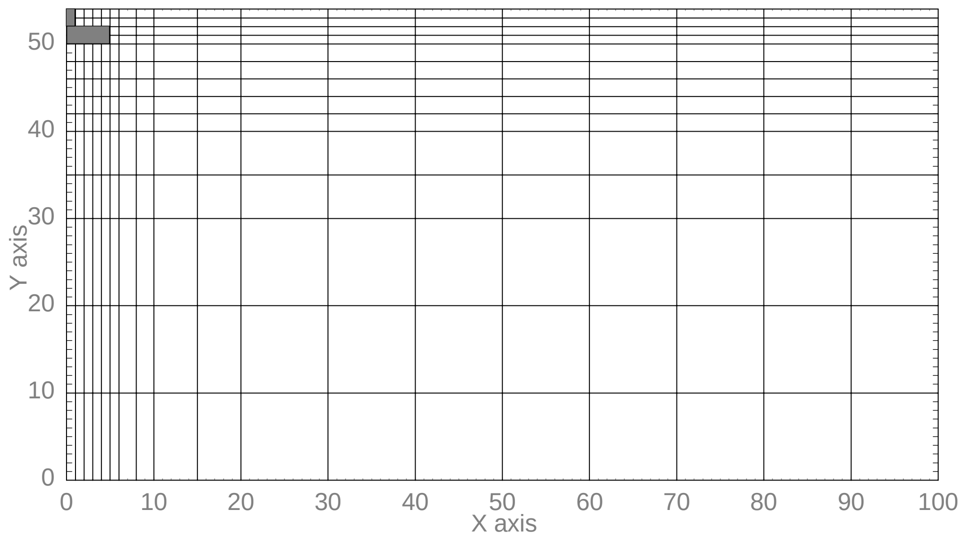
As shown in this figure, two rows of anchors were used in both subgrade cases. One row of anchors was on each side of the concrete pier (modeled) that connects the footing to the superstructure (not modeled). All simulated vertical forces were applied at the top of the concrete pier. As noted previously, both 150 kip/in^2 (1000 MPa) and 60 kip/in^2 (400 MPa) yield strength anchors were investigated. The total anchor steel area per foot of footing was varied so that the same vertical force could be applied to each combination of subgrade and anchor strength with the same safety factor against yield of the anchor steel. The steel areas used are given in Table 1.

Table 1. Anchor Areas per Foot (Metre) of Footings

subgrade	anchor area, in^2/ft (mm^2/m)	
	f_y = 150 kip/in^2 (1000 MPa)	f_y = 60 kip/in^2 (400 MPa)
rock	8.9 (18800)	22 (47000)
soil	0.36 (760)	0.90 (1900)

The basic finite-element mesh used for all analyses is shown in Figure 3. The soil footing is shown but the rock footing is similar (it is slightly thicker and narrower). Note the short pier that was modeled between the top of the footing and the surface. As noted previously, all vertical loading was applied to the top of this pier. The vertical plane of symmetry that exists along the centerline of the footing was used to minimize the size of the mesh. An arbitrary vertical boundary was assumed 100 feet (30 m) to the right of the footing centerline. Not shown in this figure are the one-dimensional interface elements that are located along all contacts between the solid elements representing the footing and those representing the soil backfill and soil or rock subgrade. The mesh boundary conditions were full fixity along the bottom and horizontal fixity only along the two sides. The top of the mesh was unrestrained.

Note: Soil-subgrade footing shown. Rock-subgrade footing similar.



Note: Both axes scaled in feet (1 foot = 305 mm).

Figure 3. Finite-Element Mesh

Load Simulation

The general procedure followed for each combination of subgrade, anchor strength and applied force direction (up or down) was identical:

- Prestressing of the anchors was simulated by simultaneously applying an upward force to the node at the top of the bar (spring) element simulating the anchor and a downward force of equal magnitude to the node on the top of the footing at the point where the anchor would eventually be connected. These two nodes had the same coordinates in the mesh but were initially independent. Thus the footing was the reaction for the anchor "jack" as would occur in reality.

- After a prestress force equal to the pre-determined design load was reached incrementally, the anchor was locked off by connecting the corresponding anchor and footing nodes for all subsequent analysis steps. Note that the typical actual sequence of proof testing an anchor to some load level in excess of the lock-off load then unloading to the lock-off load was not simulated but could have been (it was felt to be unnecessary for the goals of this study). The combinations of design loads and anchor areas were chosen so that the anchors in all cases would have a safety factor of two against steel yield under the full design load.

- An external force was then applied incrementally to the top of the footing pier. In most cases, the footing was loaded to twice the design load.

Material-Model Parameters

The stress-strain behavior of the solid elements (triangles and quadrilaterals) in *SSTIPNH* that were used to simulate the soil, rock and portland-cement concrete (PCC) footings/piers in this study is governed by the familiar hyperbolic material model. A detailed discussion of this model can be found in Clough and Duncan (1971), Duncan et al. (1980) and numerous other references. A concise summary can be found in Horvath (2000b). With proper choice of material parameters, a linear-elastic material can be modeled as well and this was done for both the rock and PCC.

The material, strength and hyperbolic model parameters used for two-dimensional (solid) and one-dimensional (interface) elements are summarized in Tables 2 and 3 respectively. They do not represent particular materials, but were chosen to be representative of the materials modeled based on data contained in Clough and Duncan (1971), Duncan et al. (1980) as well in the author's personal files. The notation used in this table is fairly standard so is not defined here. However, those seeking a detailed description can find one in Horvath (2000b) if desired.

Table 2. Finite-Element Analysis Material and Model Parameters: Two-Dimensional (Solid) Elements

material	material and strength parameters					hyperbolic-model parameters				
	γ_t, lb/ft^3 (N/m^3)	c, kip/ft^2 (MPa)	ϕ	$\Delta\phi$	K_o	K	n	R_f	K_b	m
sand	125 (19600)	0	35°	0	0.43	1000	0.5	0.9	300	0.25
rock and PCC	150 (23600)	294 (14)	0	0	0.2	2500000	0	0	1500000	0

Note: K and K_b for sand increased 50% in unloading and reloading.

Table 3. Finite-Element Analysis Material and Model Parameters: One-Dimensional (Interface) Elements

material	material and strength parameters			hyperbolic-model parameters				
	c_a, kip/ft^2 (MPa)	δ	$\Delta\delta$	K_n	K_s	K_{ur}	n	R_f
PCC-sand	0	24°	0	1000000000	20000	27000	1	0.9
PCC-rock	0	24°	0	1000000000	20000	27000	1	0.9

With regard to Table 3, in hindsight it may have been more appropriate to model the PCC-rock interface with adhesion as opposed to purely frictional. However, it is unlikely that this had much of an influence on the calculated results.

The anchors were modeled using linear-elastic bar (spring) elements. A Young's modulus of 29,000 kips/in^2 (200 GPa) was used for the steel.

NEW-MODEL VERIFICATION: NUMERICAL (FINITE-ELEMENT) ANALYSIS RESULTS

Uplift Loading

Soil Subgrade

In this case, the total anchor design load was 24 kip/ft (350 kN/m) of footing, which produced an average bearing stress of 2,400 lb/ft^2 (115 kPa) after prestressing. This load magnitude was chosen because it resulted in approximately 1 inch (25 mm) of footing settlement when the anchors were prestressed.

The upward displacement of the top of the footing pier as a result of the uplift force applied after the simulated anchor lock-off is shown in Figure 4. Note that this displacement is not absolute but relative to the post-construction position of the footing. Recall that the footings with prestressed anchors had moved downward approximately 1 inch (25 mm) as a result of prestressing. Only the results up to 100% of design load are shown as these are of greatest practical interest. The behavior of both prestressed anchors is similar. The behavior of the passive anchors is included for comparison. The high-strength passive anchor in particular exhibited significantly different behavior compared to the prestressed anchors and low-strength passive anchor.

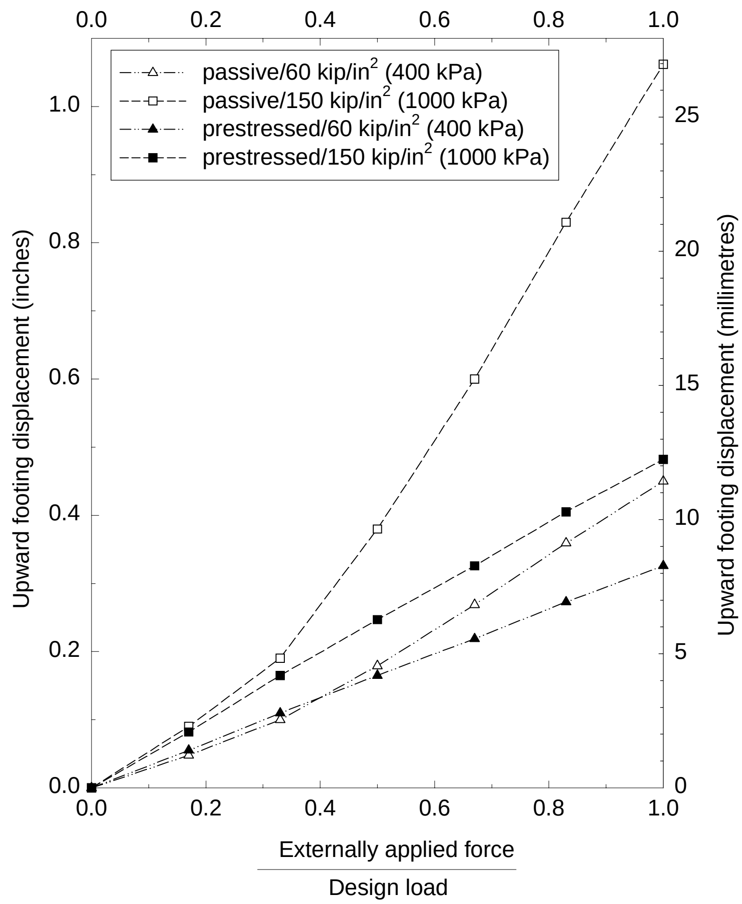


Figure 4. Uplift Load/Soil Subgrade: Upward Footing Displacements

The tensile stresses within the two prestressed anchors are plotted in Figure 5 to show the increase in stress relative to that induced initially by prestressing and lock off as the uplift force is applied. It is interesting to note that for both anchor strengths, a significant (approximately 35% to 55%) increase in tensile stress occurred by the time the full uplift design load was applied. For comparison, the limiting cases of full stress addition (labeled "upper bound") as well as no stress addition (labeled "lower bound") are shown in this figure as well. Note that the lower bound (no-stress-addition) line is the assumption of the conventional design method that was discussed previously. This method is clearly in large error on the unconservative side in this case.

The tensile stresses within the passive anchors are not shown as they are trivial, i.e. they simply increase from zero in direct proportion to the applied uplift force.

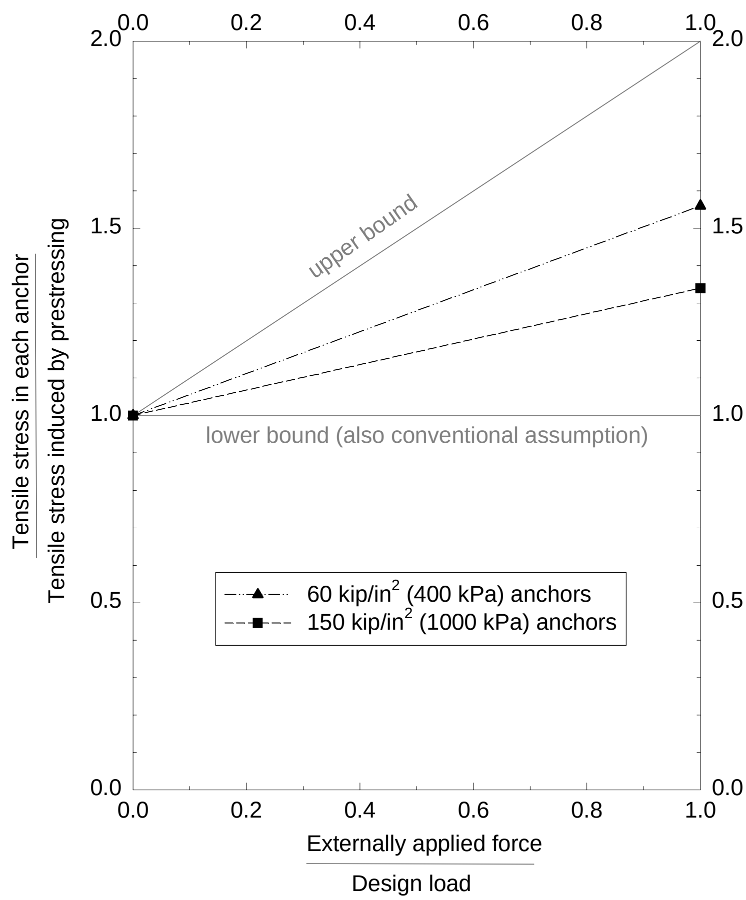


Figure 5. Uplift Load/Soil Subgrade: Stresses in Prestressed Anchors

Rock Subgrade

For the rock subgrade, an anchor design load of 220 kip/ft (10500 kN/m) was selected to produce an average bearing stress of 120 kip/ft^2 (5700 kPa) after prestressing. This is the basic maximum presumptive allowable bearing stress for hard, sound bedrock per building code in New York City where the author has considerable project experience using prestressed anchored footings on rock. The relative upward displacement of the footing under the applied post-construction uplift force is shown in Figure 6. The behavior of the two strengths of prestressed anchors is essentially identical up to the full design load, with very small magnitudes of movement relative to the passive anchors.

The tensile stresses in the anchors are not shown. Within the prestressed anchors, the stress in each case remained essentially constant (i.e. equal to that applied during prestressing) as the uplift force was applied up to the full design load. This indicates that essentially no stress increase over that induced by prestressing occurred as the uplift force was applied which corresponds to the lower-bound line shown previously in Figure 5 for the soil subgrade case. Note that this result also agrees with that assumed in the conventional design method.

The tensile stresses within the passive anchors are trivial for the same reason as discussed previously for the soil-subgrade case.

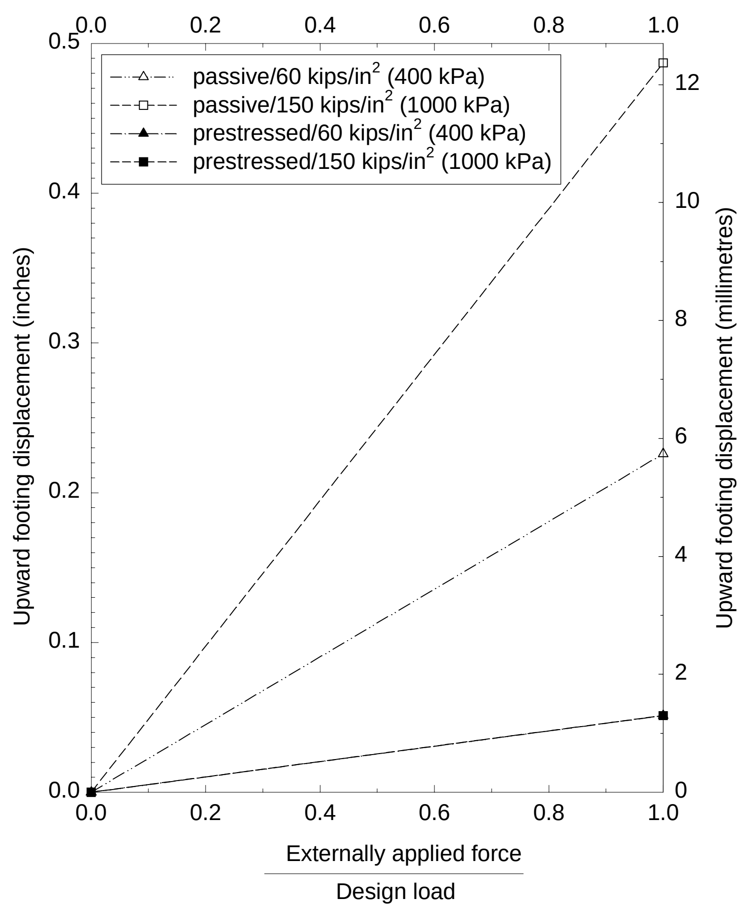


Figure 6. Uplift Load/Rock Subgrade: Upward Footing Displacements

Downward Loading

Soil Subgrade

The design load was the same as in the uplift analyses, 24 k/ft (350 kN/m), and only prestressed anchors were considered as passive anchors offer no benefit here. The relative footing settlements caused by the external force applied after prestressing are shown in Figure 7. Note that the baseline case of no anchors as well as some results beyond 100% design load are shown. As before, these settlements are not absolute but relative to the post-prestressing position of the footing (prestressing had already pulled the footing into the ground approximately 1 inch (25 mm)). The benefit of prestressing in reducing settlements under the applied force is readily apparent. Settlement reductions of the order of 30% to 60% at full design load are achieved using the anchors. It is interesting to note that the lower-strength anchor is more effective in this regard.

The rate at which the anchors destress from their initial prestress as the external downward force is applied to the footing is shown in Figure 8. Note that the anchors have a residual stress due to the initial prestress even at 100% of design load and beyond.

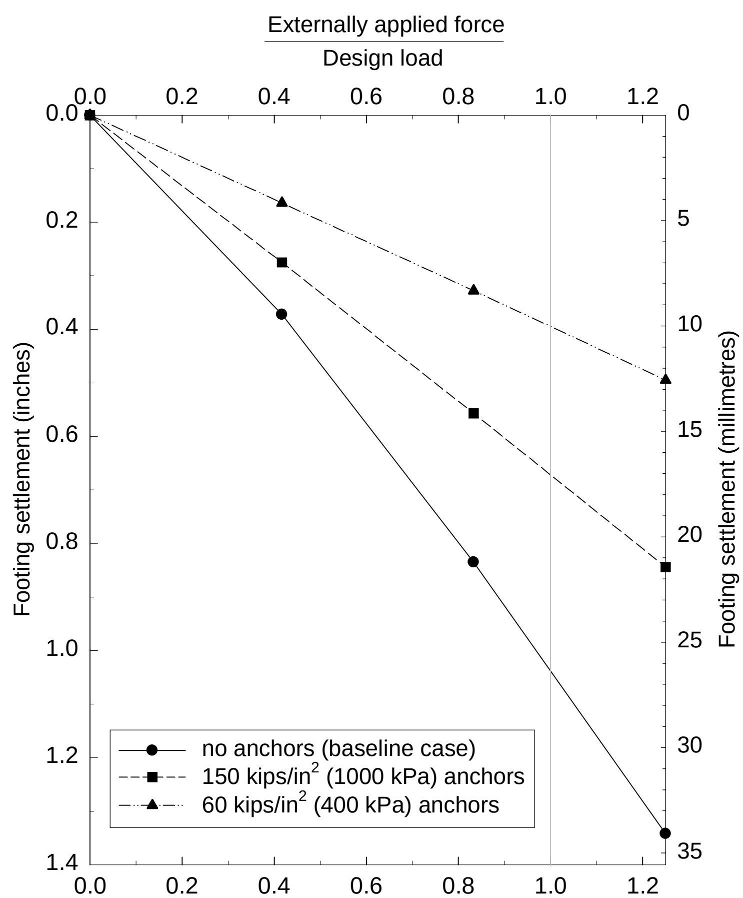


Figure 7. Downward Loading/Soil Subgrade: Footing Settlements

It is also of interest to look at the footing-subgrade contact stresses under the full design load (Figure 9). Because the anchor prestress is not fully relieved under application of the full design load, the bearing stresses are larger than for the baseline case of no anchor.

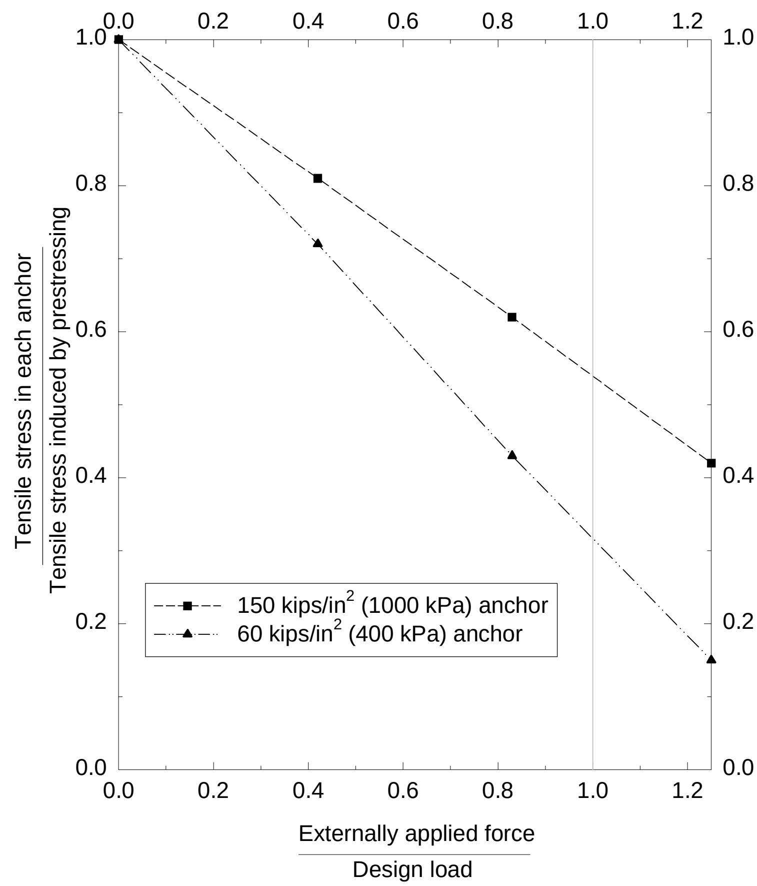


Figure 8. Downward Load/Soil Subgrade: Anchor Stresseses

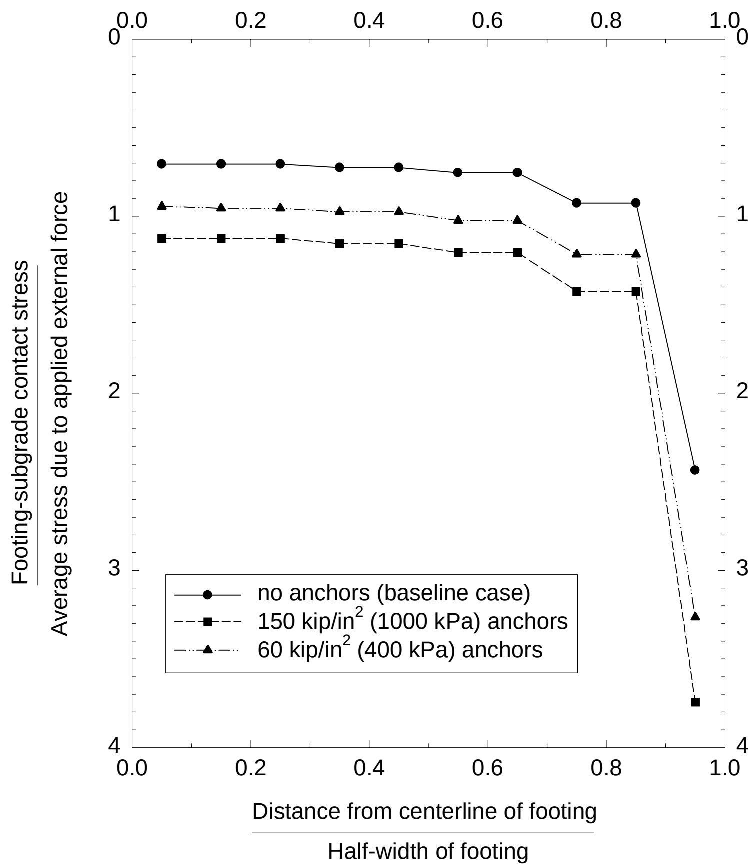


Figure 9. Downward Load/Soil Subgrade:
Footing-Subgrade Contact Stresses at Full Design Load

Rock Subgrade

The design load was the same as in the uplift analyses, 720 k/ft (10500 kN/m), and only prestressed anchors were considered. The relative footing settlements caused by the external force applied after prestressing are shown in Figure 10. As in the soil-subgrade case, these settlements are not absolute but relative to the post-prestressing position of the footing.

There was essentially no difference between the two prestressed anchors and little difference relative to the footing without anchors indicating no practical benefit from prestressing. However, the settlements are very small in magnitude to begin with so there is not much room for improvement. This is also borne out by the fact that the anchor stresses (not shown) remained essentially constant at the prestress level as the external force was applied, indicating that no stress relief in the anchors occurred because of the very small displacements. This is also evident in the footing-subgrade contact stresses which show the bearing stresses for the prestressed footing essentially doubling under 100% of the applied design load (Figure 11).

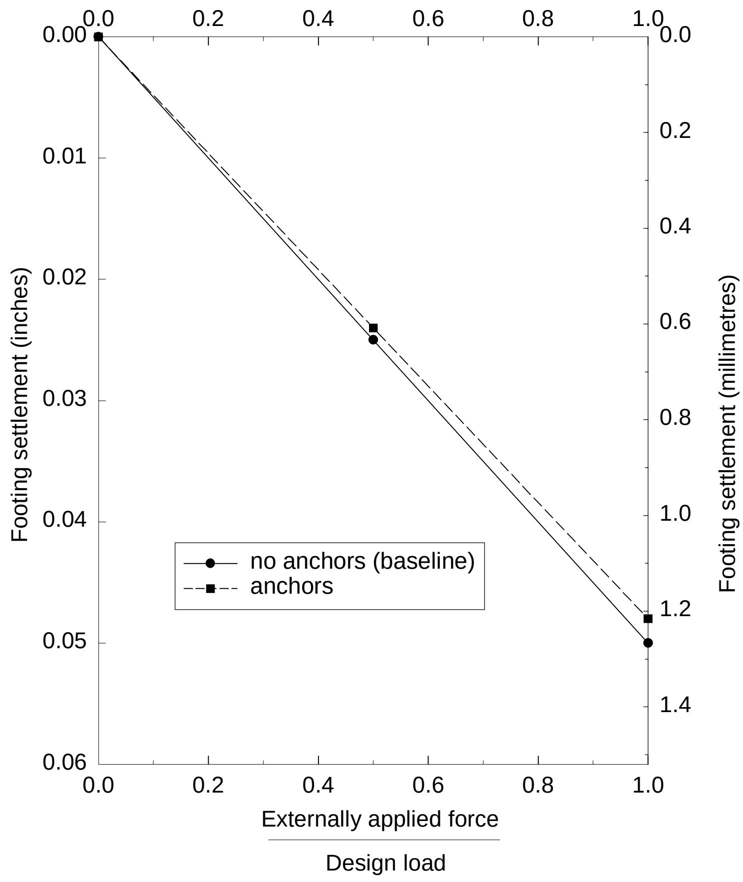


Figure 10. Downward Load/Rock Subgrade: Footing Settlement

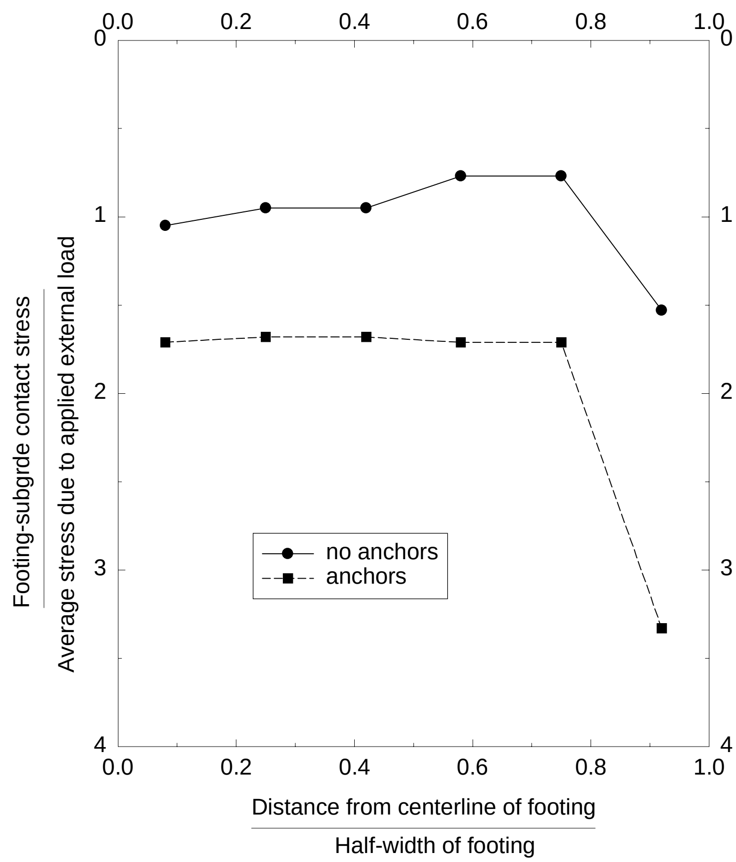


Figure 11. Downward Load/Rock Subgrade:
Footing-Subgrade Contact Stresses at Full Design Load

NEW-MODEL VERIFICATION: SPRING-MODEL RESULTS

Anchor and Subgrade Stiffnesses

It is necessary to estimate both the anchor and subgrade stiffnesses for use in the author's proposed spring model. For the anchor, stiffness is defined as the equivalent spring constant (dimensions of force per unit length) as a tensile force is applied. As noted previously, this spring has two components: elastic elongation within the unbonded length of the anchors (AE/L) plus any relative displacement between anchor and ground within the bond length. For the purposes of this report, the latter component was assumed to be zero but this is not a necessary part of the model.

For the subgrade, the stiffness was defined as some force magnitude divided by the footing displacement (uplift or settlement) that occurred under that force. The actual values were obtained from simulated load tests of the footing without anchors in the computer analyses. The resulting parameter has dimensions of force per unit length, so is a spring constant also. Note that this parameter should not be confused with the Winkler Hypothesis subgrade model that is often visualized as consisting of "soil springs". The subgrade stiffness as defined in this report is simply a secant stiffness of the overall force-displacement behavior of the footing. Again, a linear subgrade stiffness was assumed but this is not a necessary part of the model.

The calculated equivalent spring stiffnesses are summarized in Tables 4 and 5 for the anchors and subgrade respectively. Note that in this case they are per unit length (foot or metre as appropriate) of the footing. In an actual problem where the length dimension of the foundation element would be known, the spring stiffnesses would be for the entire foundation.

Table 4. Spring-Model Spring Stiffnesses: Anchors

subgrade	spring stiffness, k_A, kips/in/ft (kN/mm/m)	
	f_y = 150 kip/in^2 (1000 MPa)	f_y = 60 kip/in^2 (400 MPa)
soil	17 (10)	42 (24)
rock	1,700 (950)	4,200 (2400)

Table 5. Spring-Model Spring Stiffnesses: Subgrade

subgrade	spring stiffness, k_S, kips/in/ft (kN/mm/m)
soil	37 (21)
rock	14,000 (8300)

Example Application

To illustrate the application of the proposed spring model, the following case analyzed previously using the numerical (finite-element) solution will be re-analyzed:

- uplift loading,
- soil subgrade and
- prestressed anchor with f_y = 150 kips/in^2 (1000 MPa).

The full design uplift load for this footing is 24 kips/ft (350 kN/m/ft) which is the same as the total anchor prestress force, P^*. Substituting the appropriate parameter values from tables 4 and 5 into Equation 6 gives a relative footing upward displacement and anchor elongation of

$$w = \frac{P_F}{(k_A + k_S)} = \frac{24 \text{ kips/ft}}{17 \text{ kips/in/ft} + 37 \text{ kips/in/ft}} = 0.44 \text{ in } (11 \text{ mm}) \tag{7}$$

which compares favorably to the 0.48 in (12 mm) scaled from Figure 4.

Substituting P^*, the value of w from Equation 7 plus the total anchor spring stiffness, k_A, from Table 4 into Equation 2a gives a total anchor force of:

$$P_A = P^* + (k_A \cdot w) = 24 \text{ kips/ft} + (17 \text{ kips/in/ft} \cdot 0.44 \text{ in}) = 31.5 \text{ kips/ft } (459 \text{ kN/m}) \tag{8}$$

This value needs to be checked against the ultimate capacity of the anchors, P_{A_u}, to partially validate the calculation in Equation 7 which is based on the assumption that anchor failure (which is assumed in this report to be equal to anchor yield, not pullout) does not occur (Equation 3). P_{A_u} is assumed here to be equal to the anchor area from Table 1 (0.36 in^2/ft) times the yield strength, f_y, of the anchors (150 kips/in^2) which is 54.0 kips/ft (788 kN/m). This exceeds the value of P_A in Equation 8 which verifies that the anchor has not failed so the result in Equation 7 is tentatively valid.

Substituting P^*, the value of w from Equation 7 plus the subgrade spring stiffness, k_S, from Table 5 into Equation 2b gives a footing-subgrade contact force of:

$$P_S = P^* - (k_S \cdot w) = 24 \text{ kips/ft} - (37 \text{ kips/in/ft} \cdot 0.44 \text{ in}) = 7.7 \text{ kips/ft } (112 \text{ kN/m})$$

Because this is greater than zero, no footing lift off has occurred which completes the validation of the calculation shown in Equation 7 which is based on the assumption that footing-subgrade contact stress is maintained (Equation 4).

Checking vertical force equilibrium using Equation 1:

$$P_F + P_S - P_A = 24 \text{ kips/ft} + 7.7 \text{ kips/ft - } 31.5 \text{ kips/ft} = 0.2 \text{ kips/ft } (3 \text{ kN/m}) \cong 0 \text{ (within roundoff error)}$$

Note that the normalized anchor stress is

$$\frac{P_A}{P^*} = \frac{31.5 \text{ kips/ft}}{24 \text{ kips/ft}} = 1.31$$

which agrees well with 1.33 scaled from Figure 5. This implies that the tensile stress within each anchor under the full design load is 31% greater than that induced by prestressing. Stated another way, the stress is 31% greater than that assumed when using the conventional design method which always assumes no anchor-stress increase after prestressing. If the anchor had been designed with a safety factor of two against yield of the steel, the actual safety factor would be:

$$\frac{2}{1.31} = 1.53$$

Finally, the magnitudes of upward movement, w, to just cause the anchors to fail or the footing to separate from the subgrade can be obtained using equations 2a and 2b with w as the unknown:

$$P_A(= P_{A_u} \text{ here}) = P^* + (k_A \cdot w) \Rightarrow 54 \text{ kips/ft} = 24 \text{ kips/ft} + (17 \text{ kips/in/ft} \cdot w) \Rightarrow w = 1.8 \text{ in (45 mm)}$$

$$P_S(= 0 \text{ here}) = P^* - (k_S \cdot w) \Rightarrow 0 = 24 \text{ k/ft} - (37 \text{ k/in/ft} \cdot w) \Rightarrow w = 0.65 \text{ in (17 mm)}$$

The magnitude of *w* required to cause footing-subgrade separation (0.65 in (17 mm)) is less than that required to cause anchor failure (1.8 in (45 mm)) so governs in this case.

CONCLUSIONS

Applications Involving Uplift Loading

The results of the author's studies indicate that under uplift loading, the anchor stress always increases above the prestress level as the external uplift force is applied. This was expected and is contrary to the simple assumption of no stress change currently made in routine practice that is based solely on vertical force equilibrium and does not consider the stiffnesses of the two key problem components (anchors and subgrade).

The relative magnitude of this anchor stress increase depends primarily on the subgrade stiffness and secondarily on the anchor stiffness. For hard rock subgrades, the stress increase appears to be negligible regardless of anchor yield strength. This is because very little subgrade rebound is necessary to relieve prestress-induced stress.

However, the stress increase for soil subgrades appears to be relatively significant. This is because a significant amount of subgrade rebound is required to relieve the prestress. As a result, the tensile stress from an applied uplift force is additive to a significant extent to the prestress-induced force. As a minimum, this means that the actual safety factor against anchor yield (and bond failure for that matter) under the full design load is less than what a designer believes exists. In the extreme, the prestress and externally applied forces might combine to the extent that the anchor would begin to yield or even fail unexpectedly. Therefore, the conventional design method appears to result in a significant, unconservative error in the case of a relatively soft subgrade.

Regardless of the subgrade stiffness, a simple analytical model based on springs was presented that allows an approximate assessment of anchor and subgrade stiffness to be made in routine practice using a manual calculations if desired. Use of this model would remove much of the uncertainty or guesswork from anchor design.

An obvious question is why failures involving prestressed anchored foundations have not been observed in practice if the results implied by the numerical analyses performed by the author is correct. There are at least four reasons:

- The situation appears to be potentially critical primarily when soil or soft-rock subgrades are involved. Most prestressed anchored foundations that the author has seen involve hard-rock subgrades.

- Design values of uplift forces are generally associated with some natural, low-probability extreme event such as a "100-year" storm which probably has not occurred for the vast majority of prestressed anchored foundations. Thus the foundations have not been loaded to their design level in uplift.

- The anchors may have been designed with a safety factor against failure (by steel yield or bond failure) that is greater than two. In such a case even the upper bound of fully additive prestress and applied load would not cause failure.

- Due to a variety of reasons, the actual, as-built safety factor of the anchor against failure may be greater than that assumed in design.

Applications Involving Downward Loading

An interesting corollary issue is how footings with prestressed vertical anchors behave under downward loading. For the case of a soil subgrade, it was found that prestressing could "pre-settle" a footing to a significant extent, even if this were not the original intent of the anchors. Theoretically, by judiciously matching subgrade and anchor stiffness a footing with near-zero net settlement under externally applied downward forces could be designed. This concept might prove useful on some projects and deserves further study and verification.

Three observations are made concerning the use of prestressed anchored foundation under downward loading:

- In some applications, they may not have to have a long required service life which means that corrosion-protection measures would not be required.

- For a foundation with a large plan area, prestressing could be varied around the foundation to control any tendency to tilt due to variable structure loading and/or subgrade stiffness.

- For the case of a rock subgrade, little if any benefit occurs from using this concept. However, the settlement magnitudes of foundations on rock are rarely significant to begin with, so this is not an important issue.

Applications Involving Both Uplift and Downward Loading

The prestressed-anchor results for both types of subgrades were unexpectedly of interest in an indirect way. For both the soil and rock subgrades, it was found that the footing-subgrade contact (bearing) stress induced by prestressing was not fully relieved by downward loading, even when the full design load was reached (see figures 9 and 11 for soil and rock subgrades, respectively). In fact, for the rock subgrade (Figure 11) very little contact-stress relief occurred as the downward force was applied. For all practical purposes, the bearing stress caused by prestressing was fully additive to the bearing stress from the downward applied force so that the combined bearing stress was approximately double what might be expected.

This observation should be kept in mind when prestressed anchored foundations subjected to both uplift and downward forces (as would be common with buildings for example) are designed so that the <u>combined</u> stresses from prestressing plus load cases that produce maximum downward forces do not exceed the maximum allowable bearing stress in compression. This is a subtle point that is emphasized because, in the author's experience, most designers do not consider this issue at present. In the rock subgrade case in particular it is fortunate that the presumptive bearing values in most building codes, such as the one used in New York City with which the author is most familiar, that are the generally the basis for design are conservative. Consequently, failure to consider the fully additive nature of the bearing stresses, which may have led to the design and construction of untold numbers of theoretically underdesigned footings (some of which support buildings of substantial height), has not caused any known problems to date.

RECOMMENDATIONS

The soil-structure interaction effects of relative anchor-subgrade stiffness should be considered routinely when designing any anchored foundation. The spring model presented in this report shows promise as a relatively simple yet reasonably accurate analytical tool that can be used in routine design practice for this purpose. Observation of actual anchored foundations should be undertaken to further verify and calibrate this spring model. Especially important is investigation of relative anchor-ground displacement within the bond length, including over time, and how it effects the apparent spring stiffness of the anchor.

It is also recommended that the subgrade-foundation contact stress induced by prestressing be considered when checking bearing capacity of a foundation. Conservatively, the calculated average contact stress induced by prestressing should be added to the average stress induced by the load case producing the greatest compressive load on a foundation. Bearing capacity should be checked using the sum of these two stresses. While this may be conservative for soil or soft-rock subgrades, it appears to be quite realistic for hard-rock subgrades.

RELATED APPLICATIONS

It is of interest to note that many of the concepts described in this report have potential relevance to a different class of problems where the anchored foundation element is embedded in the ground in a more or less vertical orientation and the anchors (which may be prestressed or passive) are more or less horizontal. This includes structures such as braced excavations and anchored bulkheads. Conventional design for these structures in routine practice also tends to be based on statically determinate, force- and moment-equilibrium-only methods that ignore soil-structure interaction. This suggests that the design methods used for such structures might also benefit from improvement.

The author has performed some preliminary work along these lines over the years using a variety of relatively simple subgrade models. This experience indicates that the vertical foundation element problem is considerably more difficult to deal with than the class of problems considered in this report. This is because vertical foundation elements (walls) are usually quite flexible and thus reducing the problem to a single resultant spring for the foundation-subgrade reaction as in this report is not possible. The flexibility of the vertical foundation element (wall) and variation of ground reaction along the wall with depth must be modeled. Essentially this means many springs need to be included which becomes a daunting challenge. However, this still appears to be a fruitful line of research for the future.

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