# *Coupled Site Characterization and Foundation Analysis Research Project:*

# *Rational Selection of $\phi$ for Drained-Strength Bearing Capacity Analysis*

**Manhattan College Research Report No. CE/GE-00-1**

**by**

**John S. Horvath, Ph.D., P.E.**
**Professor of Civil Engineering**

**Manhattan College**
**School of Engineering**
**Civil Engineering Department**
**Bronx, New York, U.S.A.**

**March 2000**

Copyright by John S. Horvath.
All rights reserved.

This document may be copied and distributed for non-commercial academic and professional purposes only. However, its contents may not be altered in any manner.

Prof. John S. Horvath, Ph.D., P.E.
Manhattan College
Civil Engineering Department
Bronx, NY 10471-4098
U.S.A.

e-mail: <jhorvath@manhattan.edu>

## PREFACE

Since the 1970s, I have been pursuing a long-term research project to better integrate site characterization and geotechnical analysis for various types of foundation elements. I have termed this the *coupled site characterization and foundation analysis* concept. The primary goal of this project is to improve the predictive accuracy of foundation analyses performed in routine practice. The primary mechanism for achieving this is by incorporating the significant advances in site characterization technology that have occurred during the latter part of the 20th century into the analysis process more than is typically done at present. Where appropriate, improvements in analytical algorithms have also been made. Additional project goals are to make this integration as seamless and simple as possible to maximize appeal to geotechnical engineers in practice while still resulting in overall improvement to the state of practice.

The manuscript contained in this report is a further contribution to this project. It focuses on the common problem of performing a bearing capacity analysis for a spread footing bearing on coarse-grain soil under typical drained-strength conditions. This manuscript was recently submitted to the American Society of Civil Engineers for publication in the *Journal of Geotechnical and Geoenvironmental Engineering* as a technical note. However, given the delays and uncertainties inherent in the journal publication process this report has been prepared to make the contents of this manuscript available immediately to interested engineers.

This document may be copied and distributed for non-commercial academic and professional purposes only although its contents may not be altered in any way. Those wishing to make a hardcopy from the original PDF digital format in which this report is distributed should note that this document is set up for double-sided page printing with a side gutter for binding.

John S. Horvath, Ph.D., P.E.
Bronx, New York, U.S.A.
March 5, 2000

This page intentionally left blank.

## INTRODUCTION

Any geotechnical analysis requires a solution algorithm and input data from a project-specific site characterization assessment. The accuracy and precision of the algorithm and data should be comparable so neither dominates the accuracy and precision of the calculated results.

The evolution of geotechnical engineering through the 20$^{th}$ century has been a continual process of algorithm development and comparison of calculated results to those observed for actual structures. Whenever the accuracy of the calculations has been deficient, the tendency has been to develop improved algorithms. Although significant site characterization improvements have also occurred during the same time frame (primarily through the development of various in-situ testing methodologies), it does not appear that these improvements have been incorporated into routine practice anywhere near the extent possible.

Since the late 1980s, the pace of algorithm improvement has increased faster than ever because of the microcomputer. However, it is the author's opinion that the quality of input data development has not kept pace. As a result, the gulf between the overall quality of solution algorithms and input data is now wider than ever, especially on routine projects. Consequently, estimates of the behavior of geotechnical structures, even by experienced geotechnical engineers, still often deviate significantly from those observed (Finno et al. 1989, Briaud and Gibbens 1994).

The author's opinion is that the best way to improve predictive accuracy in geotechnical analysis should be a new initiative that focuses on improving the quality of input data used in routine practice to take advantage of the many advances in site characterization technology. Only after solution algorithms and input data are better matched in overall quality than they are at present can a more accurate assessment of the state of geotechnical predictive ability be made. This will hopefully allow a better allocation of future research resources to continue the improvement of geotechnical engineering practice in the 21$^{st}$ century.

## SCOPE AND PURPOSE OF NOTE

The author calls this overall philosophy and framework for improving geotechnical predictive accuracy *coupled site characterization and foundation analysis*. The primary purpose of this technical note is to illustrate how this concept can be applied using bearing capacity of a footing on coarse-grain soil under typical drained-strength conditions as an example. The specific methodology presented in this note represents an improved version of a procedure first published by the author in 1994 (Horvath 1994).

The contents of this note were developed as part of an ongoing long-term research project being conducted by the author into the development of methodologies for coupled site characterization and geotechnical performance analysis for both shallow and deep foundations. Previously published work related to this project has included footings (Horvath 1994), mats or rafts (Horvath 1983a, 1983b, 1989a, 1993a, 1993b, 1995), deep foundations (Horvath 1984, 1989b) and base slabs for cut-and-cover tunnels (Horvath 1993c).

## BEARING CAPACITY ANALYSIS

### Required Input Parameters

All traditional solutions for bearing capacity under drained-strength conditions neglect soil compressibility and require the same basic site information and input parameters:

- subsurface investigation sufficient to define the general stratigraphy and piezometric levels within the depth of interest,

- the effective soil unit weight (total, buoyant or a weighted average as appropriate depending on ground water conditions) within a depth equal to the footing width, *B*, below foundation level and

- a single value of the Mohr-Coulomb soil strength parameter $\phi$, the angle in internal friction, within a depth *B* below foundation level..

## Basic Concepts Concerning $\phi$

The single most important issue for bearing capacity analysis of a footing on coarse-grain soil is selection of an appropriate value of $\phi$. This is not a trivial issue as the bearing capacity factors $N_\phi$ and $N_\gamma$ are remarkably sensitive to the magnitude of $\phi$ and can vary by more than an order of magnitude for any reasonable range in values of $\phi$. Because the calculated magnitude of the gross ultimate bearing capacity, $q_{ult}$, is linearly dependent on both $N_\phi$ and $N_\gamma$, this means that $q_{ult}$ can also vary widely.

It is still quite common in routine practice for geotechnical engineers to assume that a coarse-grain soil at a given relative density has a unique value of $\phi$. Often this value is chosen from some tabulated range in a textbook or handbook. However, the first step in improving the state of practice of bearing capacity analysis is to recognize the fact that this assumption regarding $\phi$ is overly simplistic and fundamentally incorrect. In general, the stress-strain relationship for a coarse-grain soil loaded in compression will exhibit a well-defined peak stress at relatively small strain followed by strain softening until a constant stress level is reached at larger strains. This constant, large-strain stress level is referred to as the *constant volume* condition or *critical state*. Thus the soil will actually have two values of $\phi$, one corresponding to the peak strength ($\phi_{peak}$) and the other corresponding to the constant-volume strength ($\phi_{cv}$). It is generally accepted that the Mohr-Coulomb failure envelope for $\phi_{peak}$ is stress-dependent and curved concave downward while the failure envelope for $\phi_{cv}$ is stress-independent and linear as shown in Figure 1.

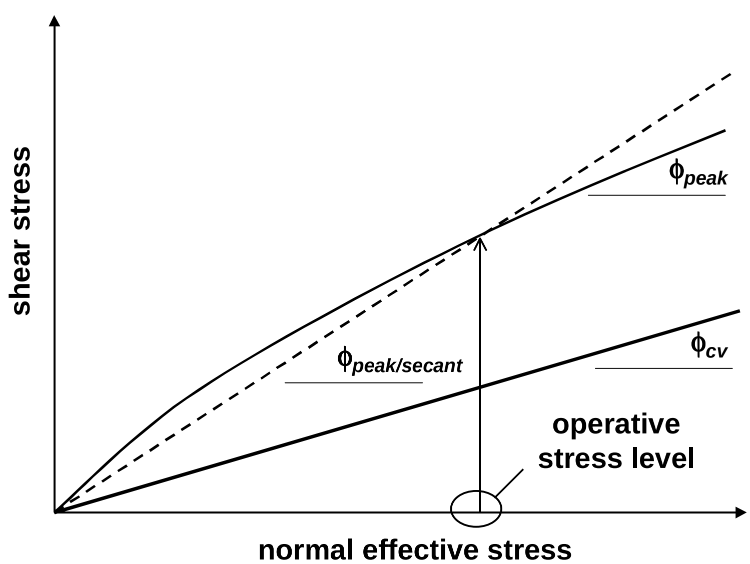


**Figure 1. Definitions of $\phi$**

The relationship between $\phi_{peak}$ and $\phi_{cv}$ is commonly expressed as

$$\phi_{peak} = \phi_{cv} + \phi_d \quad \textbf{(1)}$$

where $\phi_d$ is the *dilatancy* component of strength. Conceptually, $\phi_{cv}$ reflects the basic, inherent strength of a soil and depends primarily on the shape, size, gradation and mineralogy of the soil particles (Kulhawy and Mayne 1990). $\phi_d$ is additional strength that reflects additional soil particle interlocking that is not always present but can develop under certain conditions of confining stress and relative density (Kulhawy and Mayne 1990). In general, $\phi_d$ increases with increasing relative density and/or decreasing confining stress. For common quartz sands, $\phi_{cv}$ is typically 30° to 33° and $\phi_d$ can vary from 0° to more than 10° (Kulhawy and Mayne 1990).

### Selection of $\phi$

Clearly, the choice of an appropriate single value of $\phi$ to use in drained bearing capacity analysis is not obvious. The first decision that must be made is whether peak or constant-volume strength should be assumed. The author's opinion is that peak strength is more appropriate as constant-volume conditions would be expected to develop only at relatively large strains. The selection of peak strength is supported by the example application illustrated subsequently.

The next issue to address is how to rationally select a single value for $\phi_{peak}$. The approach adopted is to use an average value of $\phi_{peak}$ called $\phi_{peak/secant}$ as suggested by Kulhawy and Mayne (1990). The definition of $\phi_{peak/secant}$ is shown in Figure 1. Note that the key issue is to define and calculate the operative stress level that is relevant to a particular application.

## PROCEDURE FOR BEARING CAPACITY CALCULATION

### Overview

The author's procedure for calculating the bearing capacity of a footing on coarse-grain soil draws on various theoretical solutions and empirical correlations developed previously by others. The novelty of the procedure is that the site characterization for the necessary input data and bearing capacity calculation are woven into an integrated, seamless procedure that is readily solvable with standard exploration and testing methodologies and analytical tools available to any geotechnical engineer.

### Required Field Data

Ideally, data for the tip resistance, $q_c$, in kilopascals from either a cone penetrometer (CPT) or piezocone (CPTU) should be obtained as many of the calculation steps are based on $q_c$ data. However, to maximize the utility of the methodology the following procedure to develop equivalent $q_c$ data can be used if only Standard Penetration Test (SPT) field $N$-values ($N_f$) are available.

All $N_f$ data within the depth range from foundation level to a depth $B$ below foundation level must first be corrected to $N_{60}$, the $N$-value for 60% driving efficiency. This can be done using driving efficiencies actually measured during the boring process or empirical corrections based on the type of driving system used (Kulhawy and Mayne 1990). Once this is done, the equivalent $q_c$ in kilopascals for each $N_{60}$ can be estimated using the following empirical relationship given by Kulhawy and Mayne (1990):

$$q_c = N_{60} \cdot p_{atm} \cdot 5.44\,(D_{50})^{0.26} \tag{2}$$

where $p_{atm}$ = atmospheric pressure = 101 kPa and $D_{50}$ is the soil particle diameter in millimetres for which 50% of the soil is finer by weight. The value of $D_{50}$ is obtained by sieve analyses performed on soil specimens taken from within the depth range of interest using samples obtained in the SPT.

## Site Characterization

The actual or equivalent $q_c$ data are used as the primary input for a series of calculations involving several theoretical and empirical relationships. The following sequence of calculations is executed for each piece of real or equivalent $q_c$ data:

- Calculate effective vertical overburden stress, $\bar{\sigma}_{vo}$, in kilopascals at the depth of the piece of data. The appropriate soil unit weights in kilonewtons per cubic metre can be assumed based on available information (soil descriptions, sieve analyses, etc.). However, soil unit weight is sensitive to relative density, $D_r$, which is one of the parameters evaluated in this procedure. Therefore, a more rigorous approach is to include soil unit weight as an unknown in the calculation process. The dry unit weight, $\gamma_d$, can be calculated from the following theoretical relationship:

$$\frac{1}{\gamma_d} = \frac{1}{\gamma_{d\,\min}} - \left[ D_r \left( \frac{\gamma_{d\,\max} - \gamma_{d\,\min}}{\gamma_{d\,\max} \cdot \gamma_{d\,\min}} \right) \right] \tag{3}$$

  where $\gamma_{d\ max}$ and $\gamma_{d\ min}$ are the maximum and minimum dry unit weights respectively. These can be determined by laboratory testing or estimated from correlations with soil type and gradation (Kulhawy and Mayne 1990). Note that $D_r$ is expressed in its dimensionless decimal form (i.e. between 0 and 1) and is unknown initially.

- Calculate $\phi_{tc}$ which is defined as $\phi_{peak}$ in triaxial compression using the following empirical relationship given by Kulhawy and Mayne (1990):

$$\phi_{tc} = 17.6 + \left\{ 11.0 \log_{10} \left[ \frac{q_c / p_{atm}}{(\bar{\sigma}_{vo} / p_{atm})^{0.5}} \right] \right\} \tag{4}$$

- Calculate effective horizontal overburden stress, $\bar{\sigma}_{ho}$, in kilopascals using the following empirical relationship adapted from one given by Kulhawy and Mayne (1990):

$$\bar{\sigma}_{ho} = p_{atm} \left[ \frac{(q_c / p_{atm})^{1.25}}{35\, e^{(5 D_r)}} \right] \tag{5}$$

  Again, note that $D_r$ is expressed in decimal form and is unknown initially.

- Calculate $K_o$ = coefficient of earth pressure at rest using its fundamental definition:

$$K_o = \frac{\bar{\sigma}_{ho}}{\bar{\sigma}_{vo}} \tag{6}$$

This value should always be checked against the theoretical minimum and maximum values which are the coefficients of active ($K_a$) and passive ($K_p$) earth pressure respectively. Because "free-field" conditions apply here, only Rankine's theory is correct for calculating $K_a$ and $K_p$. In addition, $\phi_{ps}$ ($\phi$ under plane-strain conditions) should be used. Assuming the peak value of $\phi_{ps}$ and using information in Kulhawy and Mayne (1990):

$$\phi_{ps} \cong 1.1\,\phi_{tc} \tag{7}$$

Using Eq. 7 and assuming a planar and horizontal ground surface, Rankine's theory yields

$$K_a = \frac{1}{K_p} = \frac{1-\sin\phi_{ps}}{1+\sin\phi_{ps}} = \frac{1-\sin(1.1\,\phi_{tc})}{1+\sin(1.1\,\phi_{tc})} \tag{8}$$

- Calculate $K_{onc}$ = $K_o$ under normally consolidated conditions using the well-known empirical relationship

$$K_{onc} = 1 - \sin\phi \tag{9}$$

There is disagreement as to whether the peak or constant-volume $\phi$ should be used in Eq. 9 (Mesri and Hayat 1993, Mayne and Kulhawy 1994). Using the peak value as suggested by Mayne and Kulhawy (1994), Eq. 9 becomes

$$K_{onc} = 1 - \sin\phi_{tc} \tag{10}$$

- Calculate the overconsolidation ratio, *OCR*, using the empirical relationship given by Kulhawy and Mayne (1990) and others:

$$OCR = (K_o / K_{onc})^{(1/\alpha)} \tag{11}$$

where $\alpha = \sin\phi$. Again, there is disagreement as to whether $\phi$ should be the peak or constant-volume value (Mesri and Hayat 1993, Mayne and Kulhawy 1994). The suggestion of Mayne and Kulhawy (1994) of $\phi = \phi_{peak} = \phi_{tc}$ is adopted here so Eq. 11 becomes

$$OCR = (K_o / K_{onc})^{(1/\sin\phi_{tc})} \tag{12}$$

- Calculate $D_r$ in decimal form using a slightly simplified version of an empirical relationship given by Kulhawy and Mayne (1990):

$$D_r^2 = \left[\frac{1}{305\,(OCR)^{0.18}}\right]\left[\frac{q_c / p_{atm}}{(\bar{\sigma}_{vo} / p_{atm})^{0.5}}\right] \tag{13}$$

- Calculate $\phi_{cv}$ using

$$\phi_{cv} = \phi_{peak} - \phi_d = \phi_{tc} - \phi_d \tag{14}$$

with $\phi_d$ calculated from the following empirical relationship given by Kulhawy and Mayne (1990):

$$\phi_d = 3\left\{\left\{D_r\left[10 - \ln\left(\frac{100\,\bar{\sigma}_m}{p_{atm}}\right)\right]\right\} - 1\right\} \tag{15}$$

where $D_r$ is in decimal form and $\bar{\sigma}_m$ = mean effective stress in kilopascals which is defined here as

$$\bar{\sigma}_m = \frac{\bar{\sigma}_{vo} + 2\,\bar{\sigma}_{ho}}{3} \tag{16}$$

Eqs. 3 to 16 can be solved iteratively or explicitly by manual calculation, spreadsheet software, mathematics software or a solution-specific computer code to determine values for all parameters. The final calculated values of $\phi_{cv}$ from Eq. 14 and $D_r$ from Eq. 13 are averaged between the depth of foundation and depth of foundation plus footing width, $B$, and used as input for the bearing capacity calculation.

## Bearing Capacity Calculation

Any traditional bearing capacity theory (Hansen, Meyerhof, Vesic) can be used for calculating the gross ultimate bearing capacity, $q_{ult}$. Regardless of the theory used, the primary input variable required is $\phi_{peak/secant}$. This is determined using the average values of $\phi_{cv}$ and $D_r$ from the preceding site characterization calculations together with the following empirical equation given by Kulhawy and Mayne (1990):

$$\phi_{peak/secant} = \phi_{cv} + 3\left\{\left\{D_r\left[10 - \ln\left(\frac{100\,\bar{\sigma}_f}{p_{atm}}\right)\right]\right\} - 1\right\} \tag{17}$$

where $D_r$ is in decimal form and $\bar{\sigma}_f$ = mean effective stress in kilopascals at failure (i.e. the operative stress level shown in Figure 1) and is estimated using the following empirical equation given by De Beer (1987):

$$\bar{\sigma}_f = \left(\frac{q_{ult} + 3\,\bar{\sigma}_{vo}}{4}\right)\left(1 - \sin\phi_{peak/secant}\right) \tag{18}$$

Because Eqs. 17 and 18 are interdependent and also involve the unknown $q_{ult}$, the author has found the following iterative procedure useful:

- Initially, $\phi_{peak/secant} = \phi_{cv}$ is assumed and $q_{ult}$ calculated. This value of $q_{ult}$ is used in Eq. 18 to calculate $\bar{\sigma}_f$. This value of $\bar{\sigma}_f$ is used in Eq. 17 to calculate a revised estimate of $\phi_{peak/secant}$.

- For the second iteration, this revised estimate of $\phi_{peak/secant}$ is used to recalculate $q_{ult}$. This revised $q_{ult}$ is used in Eq. 18 to recalculate $\overline{\sigma}_f$. This revised $\overline{\sigma}_f$ is used in Eq. 17 to recalculate $\phi_{peak/secant}$.

- The revised estimate of $\phi_{peak/secant}$ from the end of the second iteration is compared to $\phi_{peak/secant}$ at the beginning of the second iteration. If they are sufficiently close in magnitude (they should be within at least 1° and preferably 0.1° in the author's opinion), the revised $q_{ult}$ calculated in the second iteration is taken to be the answer. If not, additional iterations are performed using the same sequence of calculations as in the second iteration until the desired convergence is achieved.

## EXAMPLE PROBLEM

The example chosen to illustrate the methodology outlined in this note is one of the very few well-documented case histories in the published literature of a full-scale footing loaded to a bearing capacity failure. The footing was one of five constructed and load tested for the *Spread Footing Prediction Symposium* that was a part of the ASCE *Settlement '94* conference. The author used the relevant soil and footing data that were made available to those participating in the symposium (Gibbens and Briaud 1994a, 1994b).

The particular footing chosen for this example was the "1.0 metre" footing which was a square footing with an actual width, $B$, = 991 mm. It was loaded concentrically until it settled approximately 158 mm. Thus the maximum settlement-to-footing-width ratio, $\rho/B$, was approximately 16% which is within the range of values considered necessary to define a bearing capacity failure (Vesic 1975). The footing had a depth of foundation, $D_f$, = 711 mm and thickness, $t_f$, = 1168 mm.

The following parameters based on data given by Gibbens and Briaud (1994a, 1994b) were used as input for the site characterization process (Eqs. 3 to 16):

- minimum soil dry unit weight, $\gamma_{d\,min}$ = 13.5 kN/m$^3$
- maximum soil dry unit weight, $\gamma_{d\,max}$ = 15.9 kN/m$^3$
- soil $D_{50}$ = 0.20 mm
- depth to ground water table, $z_w$ = 4.9 metres
- soil natural water content, $w_n$, within vadose zone above foundation level = 14%
- soil natural water content, $w_n$, within vadose zone below foundation level = 17%

The following results were obtained:

- average soil total unit weight, $\gamma_t$, above foundation level = 16.5 kN/m$^3$
- average soil total unit weight, $\gamma_t$, within one footing width below foundation level = 17.4 kN/m$^3$
- average soil relative density, $D_r$, within one footing width below foundation level = 61.5%
- average $\phi_{cv}$ within one footing width below foundation level = 34.0°

These results were used as input to the bearing capacity calculations using Eqs. 17 and 18 plus Hansen's bearing capacity solution (Bowles 1996). The results converged at $\phi_{peak/secant}$ = 40.0° which yielded $q_{ult}$ = 1828 kPa. Subtracting the footing dead load stress, $\overline{\sigma}_{DL}$, = 27 kPa from $q_{ult}$ yielded $q_{net}$ = 1801 kPa. The estimated force to cause bearing failure = $q_{net} \cdot B^2$ = 1769 kN. The measured force scaled from Figure 7 in Briaud and Gibbens (1994) was approximately 1690 kN. Therefore, the calculated force exceeded the measured in this case by approximately 5% which the author considers to be excellent agreement. It is of interest to note that the calculated results using $\phi_{cv}$ = 34.0° yielded a failure force = 747 kN which is less than one half of that measured. This supports the author's opinion that the use of $\phi_{cv}$ in shallow foundation bearing capacity assessments is excessively conservative.

## APPENDIX. REFERENCES

Bowles, J. E. (1996). "*Foundation analysis and design*", 5th edition, McGraw-Hill, New York, N.Y., U.S.A., 1175 pp.

Briaud, J.-L. and Gibbens, R. M. (1994). "Test and prediction results for five large spread footings on sand", *Predicted and Measured Behavior of Five Spread Footings on Sand*, J.-L. Briaud and R. M. Gibbens (eds.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 92-128.

De Beer, E. E. (1987). "Analysis of shallow foundations", Chapter 6 in *Geotechnical Modeling and Applications*, S. M. Sayed (ed.), Gulf Publishing Co., Houston, Tex., U.S.A., pp. 212-321.

Finno, R. J., Achille, J., Chen, H.-C., Cosmao, T., Park, J. B., Picard, J.-N., Smith, D. L and Williams, P. G. (1989). "Summary of pile capacity predictions and comparison with observed behavior", *Predicted and Observed Axial Behavior of Piles*, R. J. Finno (ed.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 356-385.

Gibbens, R. M. and Briaud, J.-L. (1994a). "Data and prediction request for a spread footing prediction event", *Predicted and Measured Behavior of Five Spread Footings on Sand*, J.-L. Briaud and R. M. Gibbens (eds.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 11-85.

Gibbens, R. M. and Briaud, J.-L. (1994b). "Addendums to the prediction package", *Predicted and Measured Behavior of Five Spread Footings on Sand*, J.-L. Briaud and R. M. Gibbens (eds.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 86-91.

Horvath, J. S. (1983a). "A new subgrade model applied to mat foundations", *Journal of Geotechnical Engineering*, Vol. 109, No. 12, American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 1567-1587.

Horvath, J. S. (1983b). "The modulus of subgrade reaction: a new perspective", *Journal of Geotechnical Engineering*, Vol. 109, No. 12, American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 1591-1596.

Horvath, J. S. (1984). "Simplified elastic continuum applied to the laterally loaded pile problem - part I: theory", *Laterally Loaded Deep Foundations: Analysis and Performance*, American Society for Testing and Materials, Philadelphia, Pa., U.S.A., pp. 112-121.

Horvath, J. S. (1989a). "Subgrade models for soil-structure interaction analysis", *Foundation Engineering: Current Principles and Practices*, F. H. Kulhawy (ed.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 599-612.

Horvath, J. S. (1989b). "A prediction of deep foundation capacity and settlement", *Predicted and Observed Axial Behavior of Piles*, R. J. Finno (ed.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 75-82.

Horvath, J. S. (1993a). "Subgrade modeling for soil-structure interaction analysis of horizontal foundation elements", paper presented at a special joint meeting of the Structural and Geotechnical groups of the American Society of Civil Engineers Metropolitan Section, New York, N.Y., U.S.A.

Horvath, J. S. (1993b). "Beam-column-analogy model for soil-structure interaction analysis", *Journal of Geotechnical Engineering*, Vol. 119, No. 2, American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 358-364.

Horvath, J. S. (1993c). "Cut-and-cover tunnel subgrade modeling", *Transportation Research Record No. 1415*, Transportation Research Board, Washington, D.C., U.S.A., pp. 22-31.

Horvath, J. S. (1994). "Estimation of spread-footing settlement on a sand subgrade", *Predicted and Measured Behavior of Five Spread Footings on Sand*, J.-L. Briaud and R. M. Gibbens (eds.), American Society of Civil Engineers, New York, N.Y., U.S.A., pp. 145-148.

Horvath, J. S. (1995). "Subgrade modeling for mat foundations: a review and critique of analytical methods", *Special Publication No. 152*, E. J. Ulrich (ed.), American Concrete Institute, Detroit, Mich., U.S.A., pp. 117-160.

Kulhawy, F. H. and Mayne, P. W. (1990). "*Manual on estimating soil properties for foundation design*", Report EL-6800, Research Project 1493-6, Electric Power Research Institute, Palo Alto, Calif., U.S.A., 1990.

Mayne, P. W. and Kulhawy, F. H. (1994). "The coefficient of earth pressure at rest: discussion", *Canadian Geotechnical Journal*, Vol. 31, No. 5, pp. 788-790.

Mesri, G. and Hayat, T. M. (1993). "The coefficient of earth pressure at rest", *Canadian Geotechnical Journal*, Vol. 30, No. 4, pp. 647-666.

Vesic, A. S. (1975). "Bearing capacity of shallow foundations", Chapter 3 in *Foundation Engineering Handbook*, H. F. Winterkorn and H.-Y. Fang (eds.), Van Nostrand Reinhold Co., New York, N.Y., U.S.A., pp. 121-147.