Mathematical Modeling of the Stress-Strain-Time Behavior of Geosynthetics Using the Findley Equation: General Theory and Application to EPS-Block Geofoam

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by

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EXECUTIVE SUMMARY

The time-dependent mechanical (stress-strain) behavior of polymeric geosynthetics has been a topic of significant interest for many years. Time-dependent mechanical behavior can be considered in practice using either an empirical or deterministic approach. There is a growing preference for the deterministic approach so that time-dependent mechanical behavior can be calculated explicitly in a theoretically sound manner instead of relying on empirical methodologies that utilize strength reduction factors or allowable stresses.

When using the deterministic approach in geosynthetics practice, the time-dependent component of strain has often been modeled mathematically using a power-law function, with the resulting constitutive equation referred to as the *Findley equation*. However, this terminology has almost always been applied incorrectly because the “true” Findley equation requires evaluating five material-dependent parameters and it appears that this has not been done for geosynthetics. This is a potentially important issue because the true Findley equation has proven to be sufficiently accurate for estimating the long-term behavior of a wide variety of both polymeric as well as non-polymeric composite materials in practice since the 1950s. The long-term validity of a simpler power-law-function that uses fewer material parameters compared to the Findley equation has not been demonstrated in general.

This report describes the components of the true Findley equation as well as some of the commonly used simplified equations derived from or related to the Findley equation. An illustration of how to apply the Findley equation to geosynthetics is made using compressive-creep-test data for generic block-molded expanded polystyrene (EPS-block) geofoam. EPS-block geofoam has been and is the most commonly used geofoam product worldwide. However, the presentation is completely general so the Findley equation procedure can easily be applied to traditional planar polymeric geosynthetics such as geogrids, geomembranes, and geotextiles.

Also included in this report is a comparison between the Findley equation results for EPS-block geofoam and a simpler power-law model proposed in the 1980s by the LCPC in France. The results presented raise new questions about the accuracy of the LCPC power-law model and its continued use in practice.

Section 1 - Introduction

1.1 Purpose of Report

The time-dependent (creep and relaxation[1]) mechanical (stress-strain) behavior of polymeric geosynthetic materials and products has always been of significant practical interest. Traditionally, the focus has been on tensile creep of planar geosynthetics such as geogrids, geomembranes, and geotextiles although there is increasing interest in tensile relaxation of these planar geosynthetics (Koerner et al. 1992, 1993) as well as compressive creep and relaxation for non-planar geosynthetics such as geofoam (Horvath 1995).

There are two conceptual approaches that can be used for dealing with time-dependent mechanical behavior:

- An empirical approach where, for example, a stress that is some fraction of the yield or rupture strength of a geosynthetic product is used in design. It is assumed or at least implied that if service-load stresses within the geosynthetic product are kept at or below this reduced ("allowable") stress value then time-dependent effects are either negligible or at least within some limit considered acceptable for the application. However, the magnitude of the time-dependent effects is generally unknown explicitly with this approach.

- A deterministic approach where some physical or purely mathematical model is used to explicitly calculate the time-dependent behavior. The calculated behavior is then evaluated on a case-specific basis to see if it is within some acceptable limit for that application.

Although experience indicates that either approach can produce acceptable results in practice, the author's preference as well as the perceived overall trend in geosynthetics practice is toward the deterministic approach. There are a number of reasons for this. These include the comfort of relying on theoretical rigor as well as the need to be able to explicitly calculate the time-dependent component of deformation or load as geosynthetic design pushes the technological envelope beyond that for which empirical methodologies have been developed and verified from observation in practice.

As part of this general deterministic trend in geosynthetics practice, since the 1980s there has been increasing interest in deterministic modeling of the time-dependent mechanical behavior of geofoam materials and products.[2] Unlike planar geosynthetics, the primary interest for geofoams involves compression which is the mode of loading typically encountered in geofoam applications. In particular, compressive creep is of interest in virtually all geofoam applications, both small-strain (≤ 1%) such as for lightweight fills and thermal insulation and large-strain (typically as much as 40% to 50%) for compressible inclusions.[3]

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[1] For those unfamiliar with these terms, *creep* is the component of strain or displacement that occurs with time under constant applied stress or force. It is separate from and in addition to the strain or displacement that occurs immediately upon stress or force application. *Relaxation* is the reduction in stress or force that occurs with time under a fixed, constant strain or displacement.

[2] In the earlier (1960s to 1970s) days of geofoam usage, the practice was to either ignore time-dependent effects completely (generally the result of ignorance that time-dependent effects occurred) or use an empirical approach.

[3] A discussion of the wide variety of geofoam functions and applications can be found in Horvath (1995).

1.2 Scope of Report

As a contribution to an ongoing program of development of practical models for defining the stress-strain-time behavior of geofoam materials and products, the primary focus of this report is the presentation of the results of a study performed by the author into the application of the well-known *Findley equation*. The particular geofoam material and product considered in detail and used to illustrate the concepts presented in this report is the most commonly used geofoam material, expanded polystyrene (EPS), and the most commonly used EPS-geofoam product, block-molded EPS (EPS block).

The Findley equation has been used since the 1950s as one of the primary mathematical models for the time-dependent mechanical behavior of solid polymeric as well as non-polymeric composite materials, especially under tensile creep (Findley and Khosla 1956, Findley 1960a, Chambers 1984a, Chambers 1984b, Chambers and Mosallam 1994). However, the study reported herein is believed to be the first published application of the Findley equation to a foam material of any type. This report may also be the first published application of the Findley equation to any type of geosynthetic.

An important clarification to these last two statements is that the term “Findley equation” has generally been applied incorrectly in geosynthetics practice and literature to equations that are not the “true” Findley equation. This widespread misidentification of mathematical models is identified, explored, and clarified in this report. Therefore, this report is also a contribution to mathematical modeling of the time-dependent mechanical behavior of geosynthetics in general and should appeal to a broad spectrum of geosynthetics researchers, practitioners, and manufacturers. This is because the demonstrated application of the true Findley equation is completely general and thus easily applicable to other polymeric geosynthetic products such as geogrids, geomembranes, and geotextiles.

It is beyond the scope of this report to address the issue of whether the Findley equation, some variation of it, or some derivation from it is the best stress-strain-time model for a given geosynthetic material or product in a given application. In the more than 40 years since the Findley equation was first proposed, there has undoubtedly been considerable research into other physical and purely mathematical material models for stress-strain-time behavior. An assessment of the absolute accuracy of the Findley equation and its variations and derivatives for geosynthetics is beyond the scope of this report and is left to others.

Section 2 - Creep Models

2.1 Overview

As noted previously, the primary time-dependent behavioral mode of interest for all geosynthetics is creep so it is logical to focus on models derived initially for that purpose.

With few exceptions, the basic constitutive equation for creep models has the following qualitative form:

$$e = e_o + e_c \tag{1}$$

where:

e = total strain at some time t after a stress application,
e_c = the time-dependent (creep) component of strain at some time t after a stress application, and
e_o = the immediate strain upon a stress application.

The stress application is assumed to occur instantaneously, be constant in magnitude, and permanent in duration. The primary exception to Equation 1 involves neglecting the immediate-strain component, e_o, for simplicity for those combinations of material, stress level, and time where it is negligible in magnitude compared to the creep component, e_c.

Theoretically, the immediate-strain component, e_o, can include both elastic (recoverable) and plastic (non-recoverable) sub-components. However, most publications on the subject either do not make the distinction or deal with stress levels such that the immediate strains are elastic. The latter is more common (Chambers 1984a, 1984b) because of its predominance in practice, especially in civil-engineering applications. This report assumes that e_o has only an elastic component although not necessarily linear.

With regard to the creep component, e_c, as summarized in Findley and Khosla (1956) and Findley (1960a) researchers have used a variety of relatively simple arithmetic functions of time (linear, logarithm, exponential, power law), either alone or in combination, to define the basic behavior of this aspect. These functions can:

- be an arbitrary mathematical assumption that appears to fit some set of data,
- have an arbitrary physical basis, i.e., the mathematical result of using a relatively simple, abstract physical model composed of various combinations of mechanical elements such as springs and dashpots such as proposed by Kelvin, Maxwell, etc., or
- have a rigorous physical basis deriving from some assumed rheological behavior of a specific material.

2.2 Findley Equation

In developing an equation that was used initially to define the creep behavior of polymeric materials in tension, Findley (Findley and Khosla 1956) used the basic form of Equation 1 with the assumption that

$$e_c = m\left(\frac{t}{t_o}\right)^{n_F} \quad \textbf{(2)}$$

i.e., a power-law function, where:

m = a dimensionless material parameter (defined further subsequently),
n_F = a dimensionless *Findley material parameter*,
t = time after stress application with units of hours, and
t_o = one hour (used to normalize time and non-dimensionalize the right side of Equation 2).

Therefore, Equation 1 rewritten using Equation 2 becomes

$$e = e_o + m\left(\frac{t}{t_o}\right)^{n_F} \quad \textbf{(3)}$$

The first observation with regard to Equation 3 is that the term t_o has generally been omitted in recent publications, e.g., Chambers (1984a, 1984b). Therefore, Equation 3 usually appears simply as

$$e = e_o + m\,t^{n_F} \quad \textbf{(4)}$$

It is interesting to note that it was Equation 4, not Equation 3, that actually appeared in Findley's original paper on the subject (Findley and Khosla 1956), with the more-complete version (Equation 3) appearing in a later paper (Findley 1960a).

It is of interest to note that the chronological precedence of Equation 4 and its concomitant predominance in the published literature has given rise to the comment (e.g., Chambers 1984a) that the Findley equation is dimensionally inconsistent because it appears that the second term on the right side of Equation 4 has dimensions of time while the other terms in this equation are dimensionless. However, there is no dimensional inconsistency provided that it is recognized that Equation 4 is a simplified, incomplete version of the true equation (Equation 3) and that, in fact, all terms in Equation 4 are dimensionless.

The more-important issue regarding Equations 3 and 4 is that their form has given rise to the very common, indeed almost universal, misconception that any creep equation incorporating a power-law function for the creep component of strain, e_c, is the "Findley equation." This misconception has appeared in geofoam-related publications (report dated April 25, 1989 by the Comité Europeén de Normalisation (CEN) Technical Committee (TC) 88; BASF 1992; Aabøe 1993) as well as more-general geosynthetics literature, e.g., Koerner et al. (1992, 1993). The reason that a general power-law equation is not the "true" Findley equation is that Findley (Findley and Khosla 1956) assumed specific relationships for e_o and m as follows:

$$e_o = e'_{o_F}\,\sinh\left(\frac{s}{s_{e_F}}\right) \quad \textbf{(5a)}$$

$$m = m'_F\,\sinh\left(\frac{s}{s_{m_F}}\right) \quad \textbf{(5b)}$$

where:

m'_F = a dimensionless *Findley material parameter*,

e'_{o_F} = a dimensionless *Findley material parameter*,

s_{e_F} = a *Findley material parameter* with dimensions of stress[4],

s_{m_F} = a *Findley material parameter* with dimensions of stress, and

s = the applied stress.

The appearance of the hyperbolic sine (sinh) function in Equations 5a and 5b was the result of theoretical considerations and assumptions regarding the rheological behavior of polymeric materials under tensile stress (Findley and Khosla 1956). A discussion of whether or not this is reasonably correct is beyond the scope of this report. As a further note, all five Findley material parameters (m'_F, n_F, e'_{o_F}, s_{e_F}, and s_{m_F}) were assumed to be:

1. material dependent;

2. stress, strain, and time independent (except as noted subsequently); and

3. dependent on various environmental factors such as temperature and water content of the material.

As an exception to Assumption 2, Findley (Findley and Khosla 1956) noted the possibility that these five material parameters could be stress dependent depending on whether the stress range used to evaluate material parameter values included only elastic or both elastic and plastic immediate deformations. Therefore, it is important to always clearly define the stress range over which a set of Findley material parameters is determined in addition to specifying the relevant environmental conditions, especially temperature.

Using Findley's assumptions, the expanded version of Equation 3 is

$$e = e'_{o_F} \sinh\left(\frac{s}{s_{e_F}}\right) + m'_F \sinh\left(\frac{s}{s_{m_F}}\right)\left(\frac{t}{t_o}\right)^{n_F} \quad \textbf{(6a)}$$

which is the complete, true Findley equation. Alternatively, expanding on Equation 4 produces

$$e = e'_{o_F} \sinh\left(\frac{s}{s_{e_F}}\right) + m'_F \sinh\left(\frac{s}{s_{m_F}}\right) t^{n_F} \quad \textbf{(6b)}$$

which is the simpler, abbreviated form of the true Findley equation and the form used throughout the remainder of this report.

In summary, in order for a power-law-function creep equation to be the Findley equation there must be explicitly stated the five Findley material parameters (m'_F, n_F, e'_{o_F}, s_{e_F}, and s_{m_F}) as in Equations 6a and 6b. This is not a trivial issue, as the Findley equation has been used since the 1950s and been proven to be an acceptably reliable long-term predictive tool for a wide variety of polymeric as well as non-polymeric composite materials (Findley and Khosla 1956, Findley 1960a, Chambers 1984a, Chambers

[4] Any consistent stress unit can be used in the Findley equation.

1984b, Chambers and Mosallam 1994). The long-term accuracy of a more-general power-law-function equation should not be inferred from the successful use of the Findley equation.

One interesting implication of Equations 6a and 6b is that the initial strain, e_o, is not linearly related to the applied stress, s, which means that the initial strain can be elastic but never linear elastic. However, the relationship between applied stress and initial strain will, for all practical purposes, be close to linear for values of s / s_{e_F} less than one. This was noted by Findley (1960a) and is explored further subsequently.

2.3 Simplified and Derivative Versions of the Findley Equation

2.3.1 Introduction. There are several equations to model creep that are either simplified or derivative versions of the Findley equation. Because these equations have potential use for geosynthetics, they are reviewed here beginning with some simplifications to the Findley equation.

2.3.2 Simplified Findley Equation. Using a concept suggested by Findley (1960a), Chambers(1984a) noted the following:

$$\bullet \text{ if } \left(\frac{s}{s_{e_F}}\right) \leq 1, \sinh\left(\frac{s}{s_{e_F}}\right) \cong \left(\frac{s}{s_{e_F}}\right) \quad \textbf{(7a)}$$

$$\bullet \text{ if } \left(\frac{s}{s_{m_F}}\right) \leq 1, \sinh\left(\frac{s}{s_{m_F}}\right) \cong \left(\frac{s}{s_{m_F}}\right) \quad \textbf{(7b)}$$

Assuming that Equations 7a and 7b are acceptably accurate, the Findley equation (Equation 6b) simplifies to

$$e = e'_{o_F}\left(\frac{s}{s_{e_F}}\right) + m'_F\left(\frac{s}{s_{m_F}}\right) t^{n_F} \quad \textbf{(8)}$$

Equation 8 will be referred to as the *Simplified Findley equation*. Note that the initial component of strain (the first of the two terms on the right side of Equation 8) is linear with respect to the applied stress so an initial linear-elastic behavior can be modeled by this equation.

Chambers (Chambers 1984a, 1984b) developed and used the Simplified Findley equation extensively by defining

$$E_{o_F} = \frac{s_{e_F}}{e'_{o_F}} \quad \textbf{(9a)}$$

$$E_{c_F} = \frac{s_{m_F}}{m'_F} \quad \textbf{(9b)}$$

where:

E_{o_F} = a constant, linear-elastic modulus for initial loading and

E_{c_F} = a constant modulus for the creep component.

In geotechnical-engineering terminology, E_{o_F} corresponds conceptually to the initial tangent Young's modulus, E_{t_i} , of a material obtained in a rapid-loading test.

Substituting Equations 9a and 9b into Equation 8 produces

$$e = \frac{s}{E_{o_F}} + \frac{s\,t^{n_F}}{E_{c_F}} = s\left[\frac{1}{E_{o_F}} + \frac{t^{n_F}}{E_{c_F}}\right] = \frac{s}{E_{v_F}} \tag{10}$$

where

$$E_{v_F} = \left(\frac{1}{\frac{1}{E_{o_F}} + \frac{t^{n_F}}{E_{c_F}}}\right) \tag{11}$$

and is called the *viscoelastic modulus*. Conceptually, E_{v_F} is a time-dependent secant Young's modulus that lumps together both immediate and creep behavior. As illustrated by Chambers (Chambers 1984a, 1984b), the concept of a viscoelastic modulus is useful for analyses performed using a secant-modulus approach to estimate, in a single calculation, the deformation at some particular stress level and time as opposed to performing an incremental analysis using a tangent modulus.

2.3.3 Horsley Equations. As another simplification to the Findley equation, Chambers (1984a) discussed a method attributed to Horsley in which the initial strain, e_o, is assumed to equal zero. This might be appropriate, for example, for a specific combination of material, stress level, and time such that the initial strain, while obviously greater than zero, is negligible in magnitude compared to the creep component of strain. The benefit of this assumption is that Equation 6b reduces to

$$e = m'_H \sinh\left(\frac{s}{s_{m_H}}\right) t^{n_H} \tag{12}$$

where:

m'_H = a dimensionless *Horsley material parameter*,
n_H = a dimensionless *Horsley material parameter*, and
s_{m_H} = a *Horsley material parameter* with dimensions of stress.

Note that the three Horsley material parameters (m'_H, n_H, and s_{m_H}) are identical conceptually to the corresponding Findley material parameters, but are given different notation in this report because the values obtained for a given material would differ from the corresponding Findley material parameters. This is because curve fitting of creep-test data to obtain the Findley and Horsley material parameter values would differ because of the different assumptions regarding e_o.

Chambers (Chambers 1984a) also noted that a corresponding *Simplified Horsley equation* based on an assumption identical conceptually to that in Equation 7b (which would eliminate the hyperbolic sine function in Equation 12) and viscoelastic modulus could be developed as was done for the Findley equation in Section 2.3.2. This is not explored further here.

2.3.4 General Power-Law (Pseudo-Findley) Equation. Moving on to derivatives of the Findley equation, Equation 4 can be rewritten in a conceptually identical but more-general form

$$e = e_o + mt^n \tag{13}$$

where:

m = a dimensionless material parameter and
n = a dimensionless material parameter.

Equation 13 could be called the *Pseudo-Findley equation* given its historical misidentification as the Findley equation in geosynthetics literature. However, in this report it is referred to as the *General Power-Law equation* to more-accurately describe it. Note that it is Equation 13 that has been widely used to date for defining the creep behavior of geosynthetics (including EPS-block geofoam), not the true Findley equation (Equation 6b).

With specific regard to applying the General Power-Law equation to EPS-block geofoam, the most-extensive testing and parameter evaluation to date that has appeared in the published literature was reported in Magnan and Serratrice (1989). Because of the relative obscurity of this reference, a summary of their results is given in Horvath (1995) as well as in the Appendix of this report. One of the interesting results reported by Magnan and Serratrice is that the parameter *n* was found to be stress dependent whereas Findley assumed a stress independence (the General Power-Law parameter *n* and the Findley material parameter n_F are equivalent as can be seen by comparing Equations 6b and 13).

Although use of the General Power-Law equation for geosynthetics in general and EPS-block geofoam in particular is not the focus of this report, how the parameters in Equation 13 can be determined from creep-test data will be described subsequently for the sake of completeness.

Section 3 - Application of the Findley Equation to EPS-Block Geofoam

3.1 Introduction

To illustrate the use of the true Findley equation in geosynthetics practice, a set of laboratory creep-test data for EPS-block geofoam was evaluated by the author. The tests used were performed during 1987 to 1989 by BASF AG in Ludwigshafen, Germany as part of a broad study into compressive creep of blocked-molded EPS. These creep tests extended for almost 19000 hours (2.2 years) and are believed to be the longest-duration laboratory creep tests performed to date on block-molded EPS. Up until the time of these tests, a test duration of approximately 10000 hours (1.1 years) was considered the maximum for long-term creep tests on specimens of block-molded EPS.

The apparent reason for the chosen test duration of almost 19000 hours is that the results were evaluated using the General Power-Law equation (Equation 13). As reported in BASF (1992), extrapolations using Equation 13 were judged to be acceptably accurate for only up to 30 times the duration of the creep tests from which the parameters in Equation 13 were determined.[5] Because a goal of the BASF tests was to estimate EPS-block geofoam behavior for an assumed 50-year design life of a geofoam structure[6], a minimum test duration of (50/30) years = 1.67 years = 14600 hours was required. The actual test durations of approximately 19000 hours obviously exceeded this requirement and would presumably provide a reliable extrapolation of behavior for (19000 · 30) hours = 570000 hours ≅ 65 years. It is significant to note that the interpreted results from these BASF tests were incorporated into the German national design manual for EPS-block geofoam lightweight fills for roads (*Merkblatt* 1995, BASF 1995) as well as used in ongoing CEN TC 88 studies.

3.2 Description of Test Conditions

The creep-test results for a set of three specimens of identical density (20.3 kg/m^3 (1.27 lb/ft^3))[7] were selected for the author's study. As discussed in Horvath (1995), EPS with a density of 20± kg/m^3 (1.25± lb/ft^3) was for many years the de facto standard worldwide for almost all geofoam applications, both small-strain applications (lightweight fills and thermal insulation) as well as large-strain applications (compressible inclusions). Although greater attention is now given to matching EPS density (which can be an excellent index property for geotechnically relevant parameters such as stiffness) to the intended application, 20 kg/m^3 is still a useful benchmark density for EPS-block geofoam and was judged most appropriate to use for the purposes of this study.

Each EPS specimen was a 50 mm (2 in) cube. Specimens of this shape and size are the de facto worldwide standard for testing EPS block, although variations from this are not uncommon, particularly in geotechnical laboratories where EPS-block geofoam specimens that replicate the right-circular-cylindrical

[5] The theoretical origin or justification of this is unknown to the author.

[6] This should not be interpreted as implying that EPS-block geofoam will only last 50 years in the ground. In general, the geotechnical durability of EPS-block geofoam, which is discussed in detail in Horvath (1995), is excellent and its actual longevity is unknown. The longest known in-ground use of EPS-block geofoam dates back to the 1960s. Also, the term "structure" is used in this report in the broad sense of any constructed facility and is not limited to buildings.

[7] This report adheres to typical geofoam practice worldwide where when using SI units density in kg/m^3 is quoted but when using imperial units unit weight in lb/ft^3 is quoted even though the two parameters are not strictly equivalent.

shape and dimensions of triaxial-test specimens of soil are often used. As discussed in Horvath (1995), the shape and size of EPS-block specimens can have a noticeable effect on measured results.

Each test specimen was subjected to a different stress, s, that was applied under unconfined-axial-compression conditions. Again, such test conditions are typical for creep tests on polymeric geofoam specimens. The stress magnitudes were 30 kPa (630 lb/ft^2), 40 kPa (840 lb/ft^2), and 50 kPa (1000 lb/ft^2). Stresses of these magnitudes were judged to be within the elastic range of EPS of this density, and represent the upper bound of stresses likely to be encountered in typical small-strain applications of EPS-block geofoam such as lightweight fills. Therefore, the initial-strain component, e_o, in the tests used in this study was assumed to contain only an elastic component. As noted in Section 2.1, the Findley equation can be applied to situations where the initial-strain component includes both elastic and plastic sub-components.

The laboratory environmental conditions for the duration of the tests were +23°C (+73°F) and 50% relative humidity. Again, these are de facto standard conditions and a useful benchmark for tests performed at other temperatures.

3.3 Portrayal of Creep-Test Data

There are various ways in which strain versus time data from creep tests can be portrayed. Figure 1 illustrates the most fundamental type of plot of total strain, e, versus time, t. As is well known, plots such as Figure 1 are not particularly useful for interpreting creep-test data. Figure 2 portrays the same data but using a common-log scale for time. Again, the outcome is not particularly useful nor is it if a common-log scale is used for both strain and time as shown in Figure 3.[8]

As a corollary observation, the nonlinearity of the results in Figure 2 would appear to argue against the use of a common-log arithmetic function, as opposed to a power-law arithmetic function, for defining the time dependency of the creep-strain component for EPS-block geofoam. A creep equation incorporating a common-log arithmetic function of time is apparently sometimes referred to as the *Struik equation* and has been suggested recently for EPS-block geofoam (undated report prepared for CEN TC 88).

One observation with respect to the particular set of data used in this study is that the specimen stressed at 50 kPa (1000 lb/ft^2) was apparently somewhat stiffer than the other two specimens. This is visually evident in Figures 1 and 2 as the relative spacing between the 30 kPa (630 lb/ft^2) and 40 kPa (840 lb/ft^2), and 40 kPa (840 lb/ft^2) and 50 kPa (1000 lb/ft^2) curves should be identical as time approaches zero. This is because all three stresses were within the elastic range (generally assumed to be linear elastic) for EPS block of the density tested. This variation in specimen stiffness is quantified subsequently.

In addition, if the stiffnesses of the specimens used in this study are taken as a group they appear to be toward the lower end of the range of stiffnesses expected for block-molded EPS of this density (Horvath 1995). The slight relative and absolute variation in specimen stiffness was judged to simply represent inherent slight variations that have been observed to occur for EPS-block geofoam (Horvath 1995). However, it does point out the need for careful evaluation of test specimens used for creep tests as well as the need for multiple tests at each material density and stress level so that the inevitable variations in specimen parameters are averaged out statistically.

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[8] The strain axis in Figures 1-3 is often scaled positive downward. This does not affect the stated conclusion that plots such as Figures 1-3 have limited practical use.

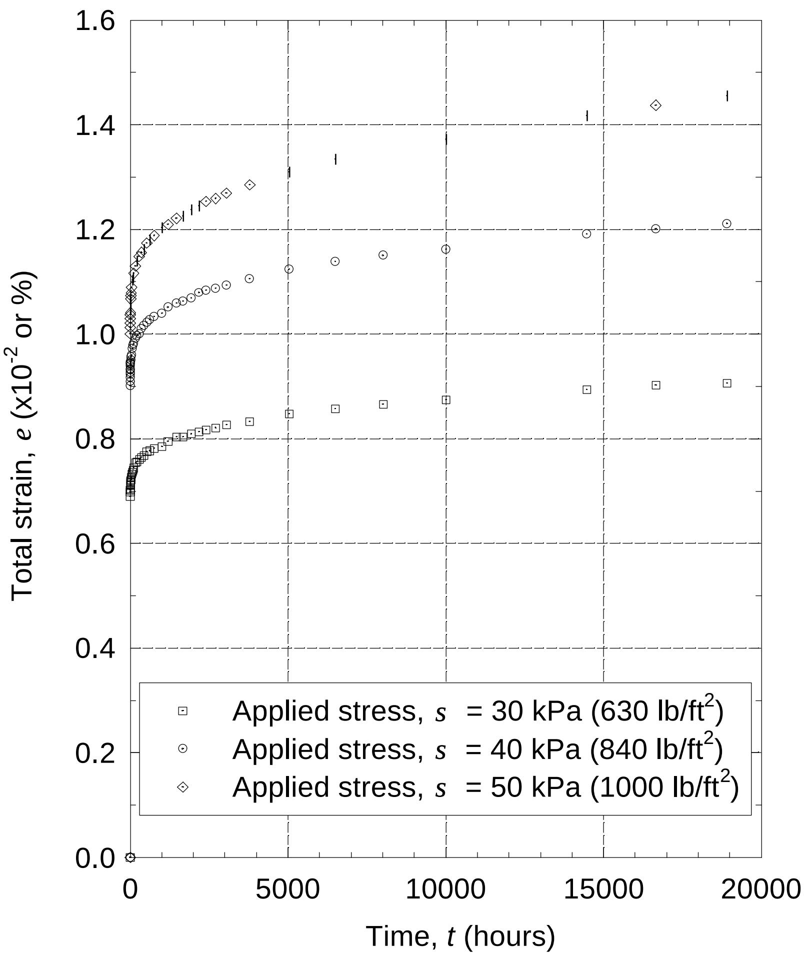

Figure 1. Creep-test results: Total strain v. time

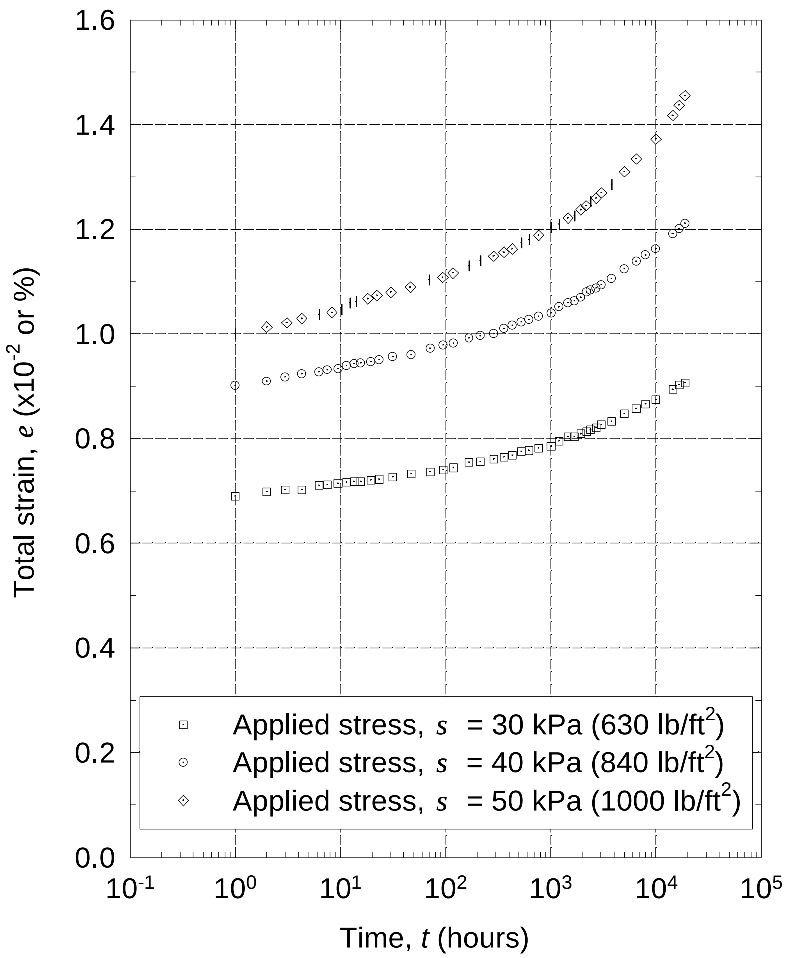

Figure 2. Creep-test results: Total strain v. $\log_{10}$ time

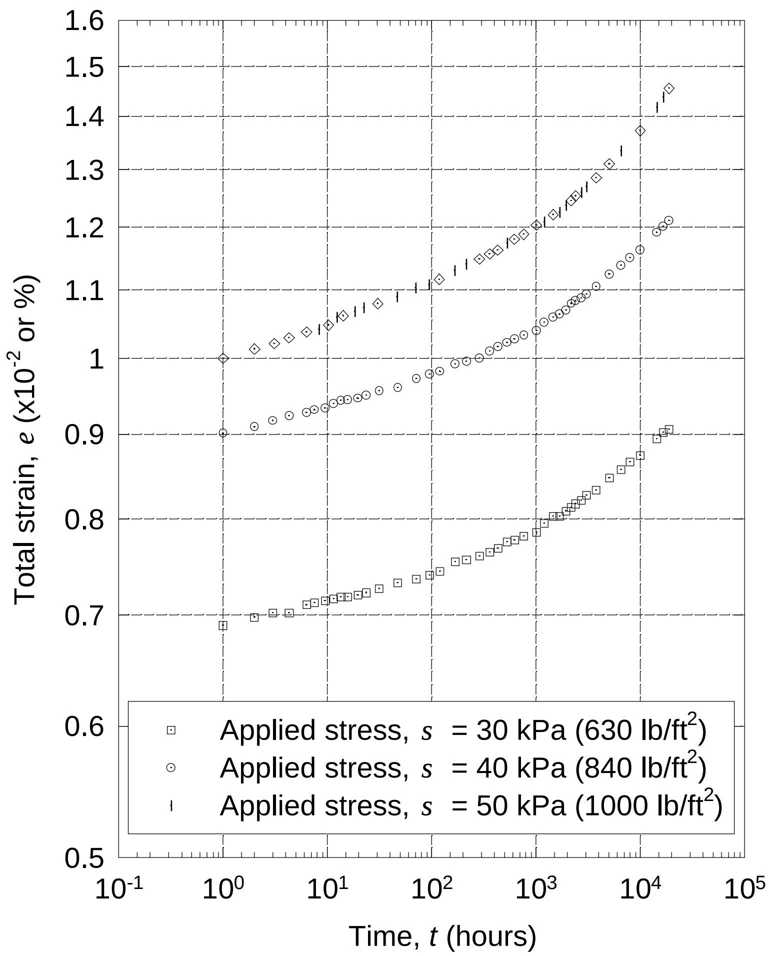

Figure 3. Creep-test results: Log_{10} total strain v. $\log_{10}$ time

As another way to portray creep-test data, Findley (Findley and Khosla 1956) suggested plotting only the time-dependent (creep) component of strain, e_c (=e-e_o), versus time, each on a common-log scale. This is shown in Figure 4 together with best-fit lines fitted to each set of data using a built-in capability of the computer software[9] with which this figure was created. The observed linear relationship is reportedly typical of polymeric materials (Findley and Khosla 1956) and the apparent reason Findley suggested this type of plot.

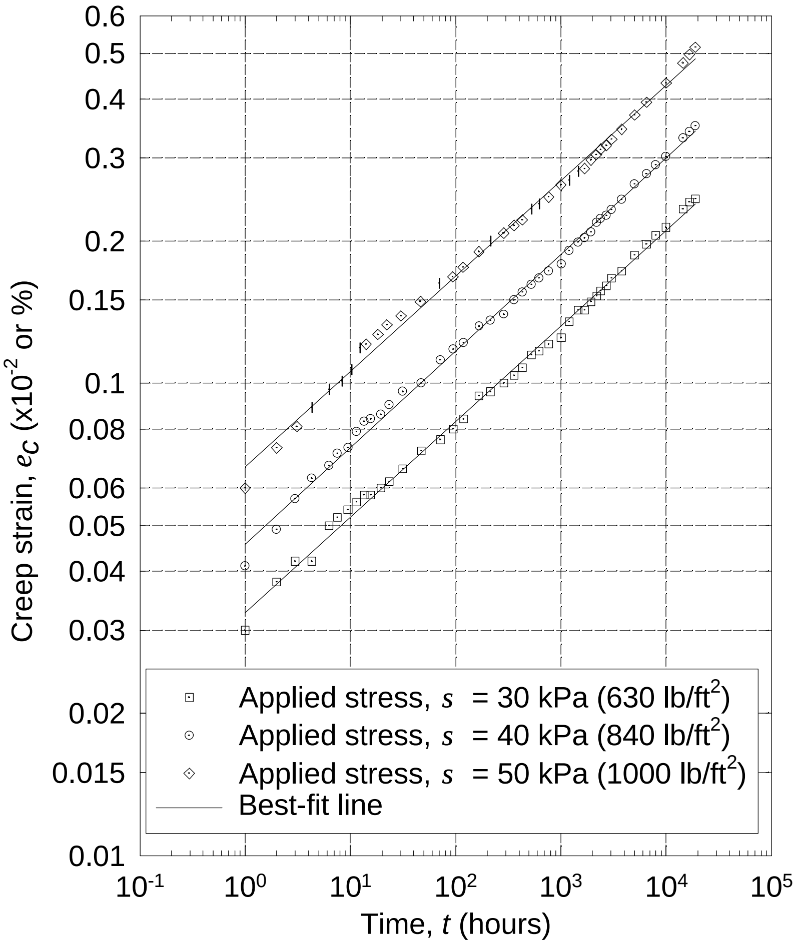

Figure 4. Creep-test results: $\mathbf{Log_{10}}$ creep strain v. $\mathbf{log_{10}}$ time

It is important to note that there is some judgment involved in creating a plot of the type shown in Figure 4, i.e., with a line fit through the test data. Findley (Findley and Khosla 1956) recommended that the primary objective always be to optimize fitting a line through all the test data and then calculating e_o afterward from the resulting best-fit line. On the other hand, Chambers (1984b) suggested that e_o always be chosen first based on

[9] SPSS *SigmaPlot® for Windows® 95, NT & 3.1*

$$e_o = \frac{s}{E_{t_i}} \quad \textbf{(14)}$$

where s is the stress applied in the particular creep test being evaluated and E_{t_i} (the initial tangent Young's modulus) is determined separately in a rapid-loading test on a specimen of material identical to that used in the creep test being evaluated. This value of e_o would then be used to fit a line through the creep-test data even if it resulted in a less-than-optimum linear fit of all the test data. Chambers reasoned that his approach was more consistent with the conceptual definition of e_o in Equation 3 as being the initial (linear-elastic as in this study) strain. Findley and Khosla (1956) also demonstrated that Equation 14 was theoretically consistent with Equation 6b. Regardless of whether the Findley or Chambers approach is used, some judgment and iteration is involved in determining e_o.

For the three tests evaluated for this study, these two philosophies (Findley v. Chambers) for fitting a line through the creep-test data yielded essentially identical results so the issue of which approach produces more-accurate results in general is moot in this case. The values of e_o determined and used by the author are given in Table 1. Also shown in this table are the values of E_{t_i} calculated using Equation 14. They confirm the qualitative observation made previously that the most highly stressed test specimen was apparently somewhat stiffer than the other two. The author recommends that for future creep tests, careful attention be paid to obtaining strain data immediately upon stress application and frequently for the first hour or so thereafter to maximize the accuracy in determining e_o (this was apparently not done for the tests used in this study and it complicated the curve-fitting process). Also, a companion standard rapid-loading test should always be performed on a specimen of the same sample of material used for a creep test.

Table 1. Initial strains used in Findley equation and test-specimen moduli

Applied stress, s	**Initial strain, e_o**	**initial tangent Young's modulus, E_{t_i}**
30 kPa (630 lb/ft^2)	0.0066	4500 kPa (94 k/ft^2)
40 kPa (840 lb/ft^2)	0.0086	4700 kPa (98 k/ft^2)
50 kPa (1000 lb/ft^2)	0.0094	5300 kPa (110 k/ft^2)

3.4 Determination of the General Power-Law Equation Parameters

Although the General Power-Law equation is not the primary focus of this report, its widespread use in practice makes it of some interest to discuss at this point. Figure 4 is all that is needed if only the parameters for the General Power-Law equation (Equation 13) are desired. For each applied stress, the parameter m is the magnitude of creep strain at a time 10^0 (= 1) hour and the parameter n is the slope of each best-fit line. The General Power-Law equation parameters for the tests shown in Figure 4 are summarized in Table 2.

Table 2. General Power-Law equation parameters

Applied Stress, s	***m***	***n***
30 kPa (630 lb/ft^2)	0.000327	0.202
40 kPa (840 lb/ft^2)	0.000457	0.204
50 kPa (1000 lb/ft^2)	0.000666	0.202

It should be noted that the most-accurate way to determine the General Power-Law equation parameters for use in practice is to perform at least one creep test for each stress level of interest for each material (density of EPS-block geofoam in this case) of interest. This is because *m* is stress-level dependent for a given material and each EPS density effectively represents a different material. However, given a sufficient number of tests over a range of EPS densities and applied stresses of interest, empirical relationships can be developed to allow interpolated calculation of the General Power-Law equation parameters *m* and *n* for EPS density and stress combinations within the ranges studied but for which explicit creep tests were not performed. Just such a study was performed in France during the 1980s and reported in Magnan and Serratrice (1989). However, as discussed in Horvath (1995), there are important practical limits and qualifications on the parameter relationships given by Magnan and Serratrice as their work focused on small-strain applications for lightweight-fills (as did the BASF tests used in this study).

It is of interest to note that the values of *n* in Table 2 are, for all practical purposes, constant with stress as Findley suggested they should be. This is at odds with earlier findings for EPS block as reported by Magnan and Serratrice (1989) who reported a stress-level dependency for *n*. This is issue explored further in the Appendix of this report.

3.5 Determination of the Findley Equation Parameters

Unlike for the General Power-Law equation, only one set of Findley equation parameters is required and determined for a given material (a specific EPS-block geofoam density in this case) and overall stress-level range of interest because the Findley equation inherently incorporates the effect of applied stress variation. Theoretically, determination of the Findley-equation parameters requires only two creep tests on the same material, with a different stress level applied in each test to bracket the overall stress-level range of interest. The results from three tests were available for the study described in this report and the author considers this the practical minimum with more than three tests highly desirable to allow for the inevitable scatter in data due to inherent manufacturing variations in material properties as was encountered in the series of tests used for this study.

To begin with, the Findley parameter n_F is the same as the General Power-Law parameter *n* as noted previously in Section 2.3.4. Therefore, n_F is the average slope of the best-fit lines in Figure 4. These results were already summarized in Table 2, so for the tests considered in this study n_F = 0.20. Again, note that n_F is essentially constant for a given material (EPS-block geofoam density in this case) and stress-level range as Findley suggested it should be.

Determination of the next two of the remaining four Findley parameters requires equating ε_o in Equation 5a with the results in Table 1 to solve for the two unknowns, ε'_{o_F} and σ_{ε_F}. Findley (Findley and Khosla 1956) originally suggested a trial-and error graphical approach although a numerical approach using computer software to find best-fit parameters is easier nowadays. Regardless of the method used, Chambers (1984b) cautions that this parameter-value determination should be done with some care, especially when minimal sets of data are available, because of the nonlinearities introduced by the hyperbolic sine function involved. Based on experience with this study, the author confirms Chambers caution as being appropriate. The author obtained the following results for this study using computer software to fit the lines:

- ε'_{o_F} = 0.0110
- σ_{ε_F} = 54.2 kPa (1130 lb/ft^2).

A useful guideline for evaluating the "correctness" of these two Findley material parameters for EPS-block geofoam is that $\sigma_{\varepsilon_F} / \varepsilon'_{o_F}$ should be close to the value of E_{t_i} as suggested by Chambers (1984b).

In this case, s_{e_F} / e'_{o_F} = 4930 kPa (103 kip/ft^2) which is consistent with the values shown in Table 1 and the low-end of the range of values for this EPS density as reported in Horvath (1995). As additional guidelines for evaluating parameter-value correctness, the author suggests that e'_{o_F} should approximately equal the elastic-limit strain for EPS block (generally taken to be 1% (= 0.01) as discussed in Horvath (1995)) and s_{e_F} should equal the corresponding elastic-limit stress for EPS (which would be in the range of 50 kPa (1000 lb/ft^2) to 60 kPa (1300 lb/ft^2) for EPS of this density). Note that both of these additional criteria are met here.

Determination of the last two Findley material parameters requires using Equation 5b together with the results in the first two columns of Table 2 to solve for the two unknowns $m¢_F$ and s_{m_F}. Again, Findley (Findley and Khosla 1956) originally suggested a trial-and error graphical approach and Chambers (1984b) made the corresponding caution that the author found to be valid. The author obtained the following results for this study based on using a computer-based numerical-solution approach:

- $m¢_F$ = 0.000305
- s_{m_F} = 33.0 kPa (689 lb/ft^2).

Judging the correctness of these latter results is more difficult than for the initial-strain component because of the lack of either a theoretical or intuitive basis as to what these two creep-component parameters should represent for EPS block. However, it is perhaps of use to note that the magnitude of the stress term, s_{m_F}, corresponds approximately to the magnitude of sustained stress below which creep strains for EPS of this density would be very small and negligible[10] even in small-strain applications such as lightweight fills and thermal insulation. The author is not aware of any rational interpretation of the parameter $m¢_F$ that might be useful in assessing the correctness of the value obtained. However, at least for this study it is of the same order of magnitude as the General Power-Law equation parameter *m* as summarized in Table 2.

Another incidental observation for the Findley parameters obtained in this study is that $s / s_{e_F} < 1$ but $s / s_{m_F} > 1$ for all applied stresses in the tests used for this study. This suggests that the Simplified Findley equation (Equation 8) discussed in Section 2.3.2 would not be appropriate for analyses involving EPS-block geofoam, except perhaps under relatively very small stresses (<30 kPa (630 lb/ft^2) in this case), because the error introduced assuming Equation 7b is valid may be excessive. This is illustrated further in Section 3.6 of this report.

In summary, the Findley material parameters determined for this study were:

- $m¢_F$ = 0.000305
- n_F = 0.20
- e'_{o_F} = 0.0110
- s_{e_F} = 54.2 kPa (1130 lb/ft^2)
- s_{m_F} = 33.0 kPa (689 lb/ft^2).

[10] As discussed in Horvath (1995), this criterion is generally taken to be one-half of the elastic-limit strain, i.e., a strain of 0.5% (= 0.05), which corresponds to one-half of the elastic-limit stress, i.e., approximately 25 kPa (520 lb/ft^2) to 30 kPa (630 lb/ft^2) for EPS-block geofoam of the density used in this study.

The corresponding Findley equation is obtained by substituting these values into Equation 6b which results in

$$\varepsilon = 0.011\sinh\left(\frac{\sigma}{54.2}\right) + 0.000305\sinh\left(\frac{\sigma}{33.0}\right)t^{0.20} \qquad \textbf{(15)}$$

The author cautions that Equation 15 is appropriate only for EPS-block geofoam:

- with a density of approximately 20 kg/m^3 (1.25 lb/ft^3),
- under an applied stress, s, with SI units of kilopascals and magnitude ≤ 50 kPa,
- with time, *t*, with units of hours and magnitude > 1, and
- at a temperature of approximately +23°C (+73°F) (or less in which case the results should be somewhat conservative).

3.6 Using the Findley Equation for Predicting Creep and Creep-Related Behavior

Equation 15 can be used to develop a variety of predictive plots within the limitations outlined at the end of the preceding section. The obvious first choice is to calculate total strains beyond the duration of the creep tests. This is shown in Figure 5 by the solid curves. Also shown in this figure are the measured test data from Figure 2. The calculated strains were intentionally limited to a time projection 30 times the test duration (approximately 570000 hours ≅ 65 years in this case) although, theoretically, there is no known limit on projections made using the true Findley equation.

The agreement between the estimated behavior using Equation 15 and the actual measured results is good, with some deviation because only one set of Findley parameters is obtained for a given set of creep-test data which may include test specimens that exhibit some physical variation between them. As noted previously in Section 3.3, such was the case here as there was a slight variation in stiffness within the initial elastic range between the three test specimens used in this study (see Table 1). This highlights the desirability of using as many specimens as possible of a given material when developing parameters for creep equations so that the impact of a particular inherent material variation is smoothed out.

Also illustrated in Figure 5 using dashed curves is the total strain calculated using the Simplified Findley Equation (Equation 8). The agreement with the actual data is not as good as for the Findley equation, especially with increasing time. This is because, as noted in Section 3.5, the assumption stated in Equation 7b, which affects the creep component of strain calculated using the Simplified Findley equation, is not satisfied for the stress levels used in this study. As a result, the error in using the Simplified Findley equation tends to increase with time in this application.

Another type of plot developed using the Findley equation (Equation 15) is shown in Figure 6. This type of plot illustrates isochronous stress-strain curves.[11] Shown in this particular plot are the calculated results for four time intervals differing by orders of magnitude plus the measured results for the first three of the four intervals. There is good agreement between the values calculated using the Findley equation and the measured values, except for the highest stress level (50 kPa (1000 lb/ft^2)) where agreement is less satisfactory. Again, this is apparently due to the variation in specimen stiffness for the set of tests used for this study.

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[11] For those unfamiliar with such curves, they represent estimated stress-strain behavior for different assumed durations of applied stress.

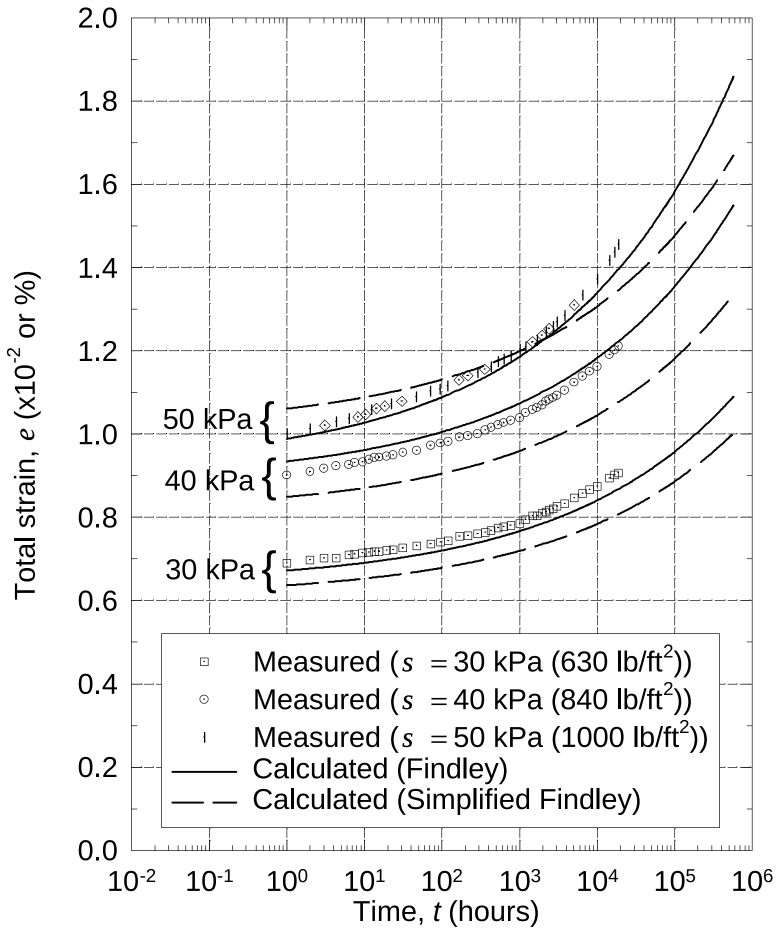

Figure 5. Comparison of measured versus calculated total strain v. $\log_{10}$ time

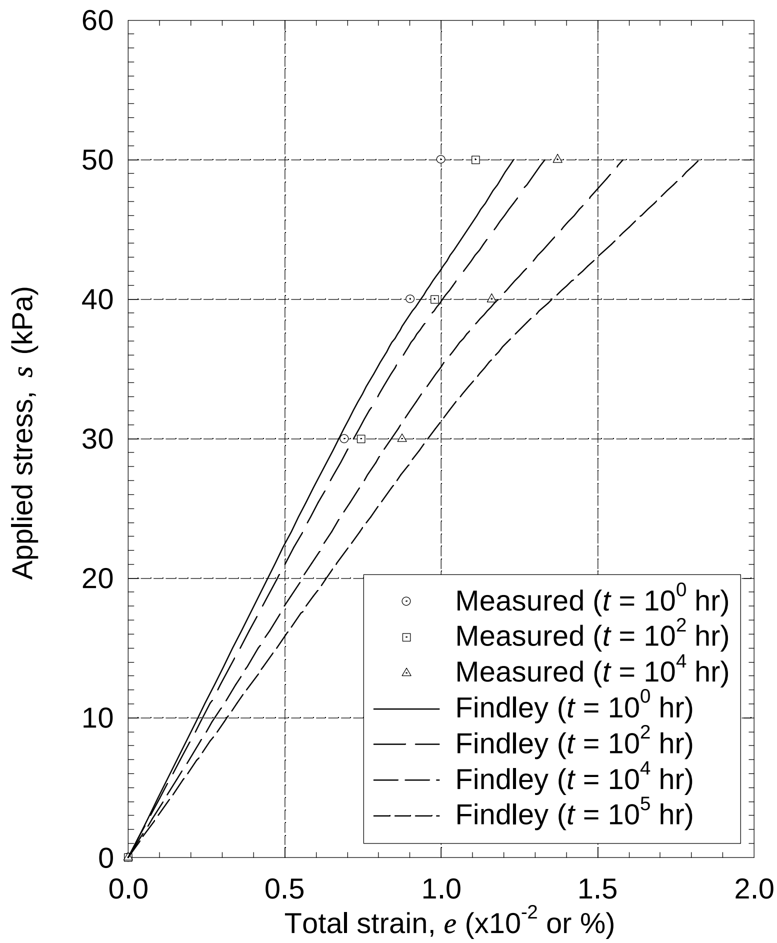

Figure 6. Isochronous stress-strain curves

The final type of creep plot explored in this report is the *Sherby-Dorn* plot. This type of plot portrays the log of total creep rate (the first derivative of creep with respect to time), $\dot{e}$, in two variants, versus either creep strain, e_c, or common-log of time, t. The utility of Sherby-Dorn plots is illustrated qualitatively in Figure 7 for the more-common creep-strain plotting variant.

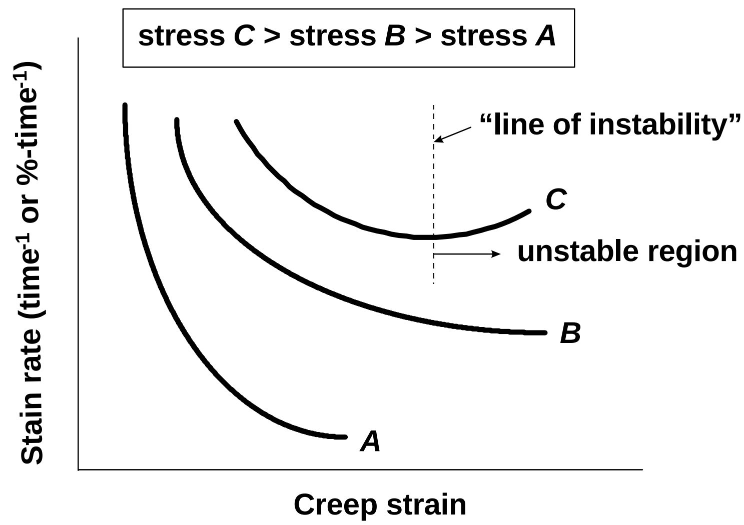

Figure 7. Qualitative example of Sherby-Dorn plot (creep-strain variant)

For a given magnitude of applied stress, one of only three behavioral categories will exist:

- The curve for stress *A* illustrates a long-term stable condition where the creep rate continues to decrease with time.
- The curve for stress *B* illustrates a transitional but still long-term stable condition where the creep rate eventually becomes constant with time.
- The curve for stress *C* illustrates a potentially long-term problematic situation where the creep rate increases at an accelerating rate at some time after a period of decreasing creep rate. This phenomenon is called *tertiary creep* and, in tensile loading cases, results in what is called *creep rupture* where the material fails suddenly at some point in time after appearing to be stable. The existence of tertiary creep is one reason why creep tests of relatively short duration can give misleading and potentially dangerous insight into long-term creep behavior as the strain rates for curve *C* appear to be deceptively stable initially, i.e., similar to curves *A* and *B*, and can lead to a false assumption that a long-term stable situation exists when it does not.

Figs. 8a and 8b illustrate the two variants of Sherby-Dorn plots for the measured data (shown by symbols) used for this study. It can be seen that the creep rates for all three tests appear to be of the Curve *A* (Figure 7) variety as would be expected for the relatively low stress levels imposed in these tests.

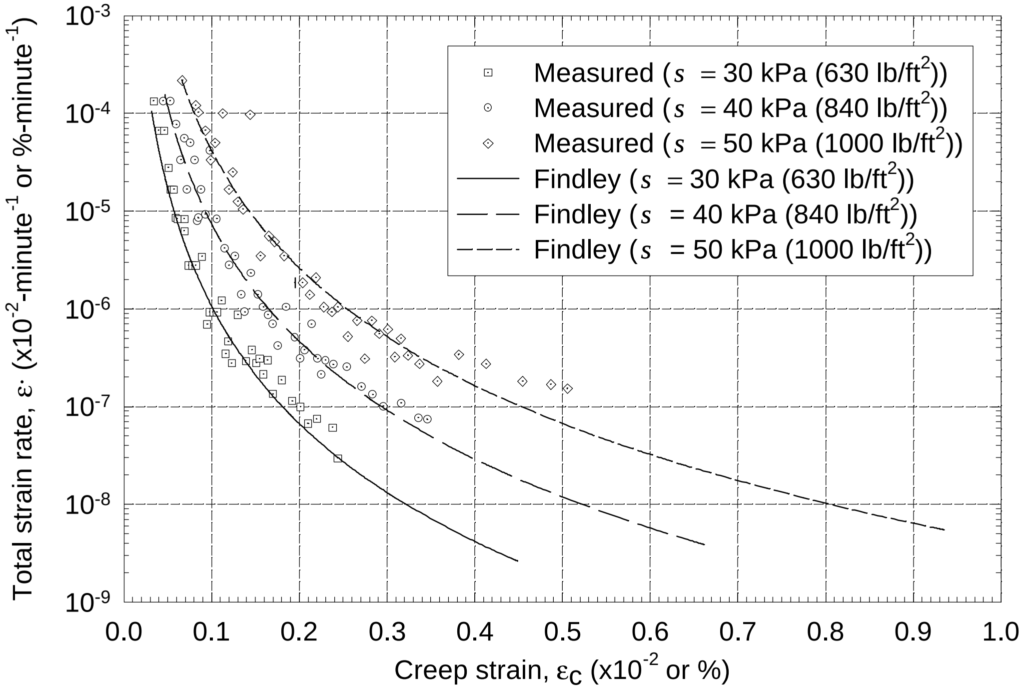

Figure 8a. Sherby-Dorn plot (creep-strain variant)

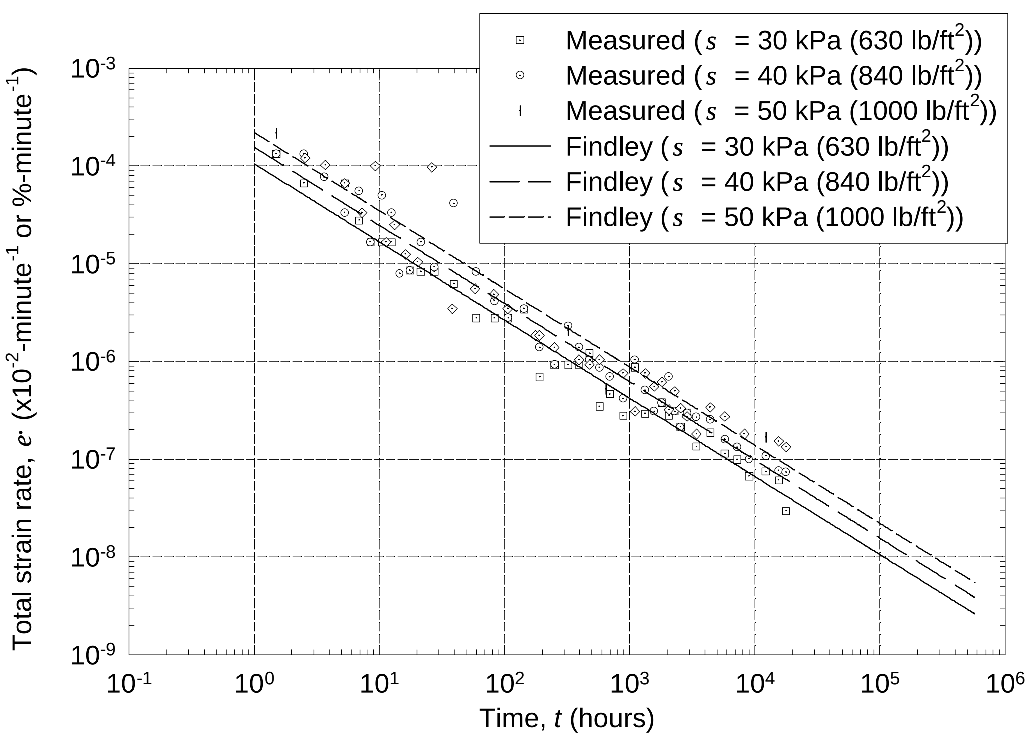

Figure 8b. Sherby-Dorn plot (log-time variant)

The Findley equation can be used to generate curves for Sherby-Dorn plots by differentiating Equation 6b with respect to time. The resulting equation is

$$\dot{\varepsilon} = m'_F \; n_F \; \sinh\left(\frac{\sigma}{\sigma_{m_F}}\right) t^{(n_F - 1)} \qquad \textbf{(16)}$$

Equation 16 was evaluated using the Findley parameters determined for this study. The results are also shown in Figures 8a and 8b using curves and show acceptable correlation with measured data.

One caution with using Equation 16 is that the Findley equation will never predict tertiary creep behavior (Curve *C* in Figure 7) because this phenomenon is simply not reflected in the mathematical assumptions used to produce Equation 6b. Therefore, estimates made using Equation 16 should never be used as a substitute for long-term creep tests to determine if tertiary creep is a potential problem for a given geosynthetic product and stress level.

3.7 Using the Findley Equation for Predicting Relaxation Behavior

The phenomenological reciprocity between creep (increasing strain under constant stress) and relaxation (decreasing stress under constant strain) for polymeric materials was noted by Findley (1960b) and Chambers (1984a). This concept was explored for traditional planar polymeric geosynthetics by Koerner et al. (1992, 1993).

As shown in Findley (1960b), Equation 6b can be rearranged as follows for relaxation

$$t = \left\{ \frac{\varepsilon - \left[\varepsilon'_{o_F} \; \sinh\left(\frac{\sigma}{\sigma_{e_F}}\right) \right]}{m'_F \; \sinh\left(\frac{\sigma}{\sigma_{m_F}}\right)} \right\}^{\frac{1}{n_F}} \qquad \textbf{(17)}$$

As noted in Chambers (1984a), for relaxation calculations σ is actually the dependent variable and *t* is the independent variable which is the reverse of that implied by Equation 17. Therefore, a relaxation equation should be expressed with σ as the calculated result, not *t* as in Equation 17. However, Equation 17 represents a compromise for arithmetic simplicity because of the hyperbolic sine functions involved.

Equation 17 has been found to produce good correlation with actual tensile relaxation test data for a variety of polymeric and composite materials (Findley 1960b). Therefore, Equation 17 was evaluated for EPS-block geofoam using the parameters determined for this study for three arbitrary small-strain levels that were consistent with the stress-level range over which the parameters were obtained. The results are shown in Figure 9. The absolute accuracy of the curves shown in this figure can only be determined once actual relaxation tests on EPS are performed. However, based on preliminary, unpublished relaxation tests performed several years ago by the author on other specimens of block-molded EPS the trends in Figure 9 appear to be correct for EPS-block geofoam strained within the elastic range.

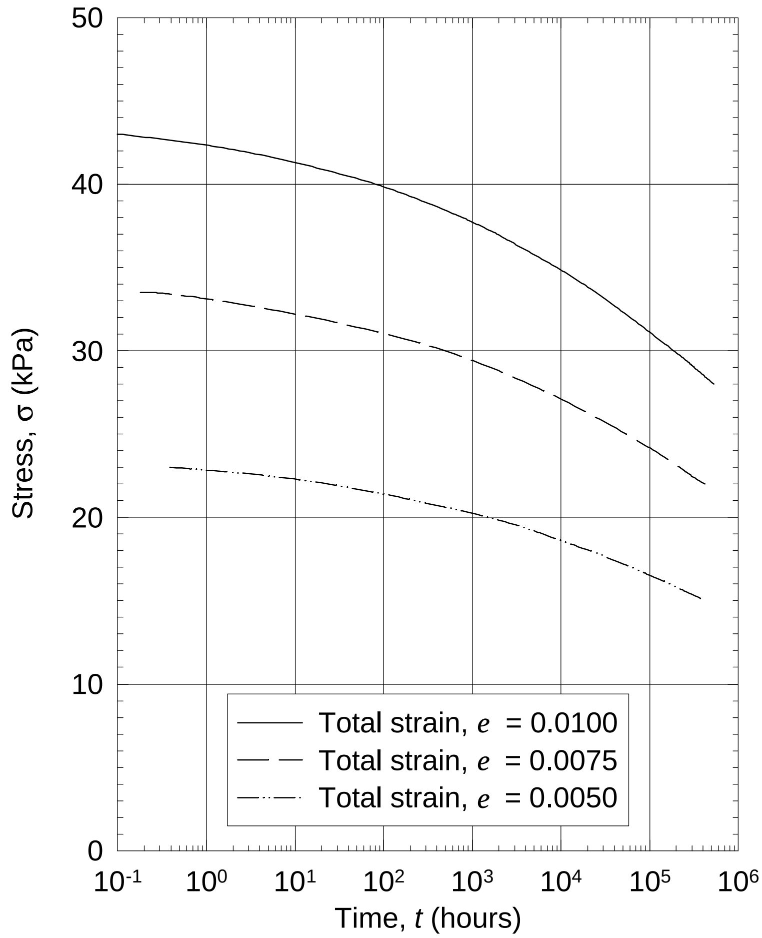

Figure 9. Calculated relaxation behavior using the Findley equation

Section 4 - Closing Comments

4.1 Summary and General Recommendations

This report has illustrated how the true Findley equation parameters can be evaluated for and applied to EPS-block geofoam geosynthetic. However, the presentation and procedures involved are completely general so the Findley equation can easily be applied to other, traditional (planar) polymeric geosynthetics such as geogrids, geomembranes, and geotextiles.

A benefit of using the Findley equation (Equation 6b) as opposed to the General Power-Law equation (Equation 13) which, to date, has been used almost exclusively for geosynthetics is that fewer creep tests are required to obtain the material creep parameters over a stress-level range of interest. In addition, the Findley equation has, in general, been verified for a wide range of polymeric and non-polymeric materials over more than 40 years. A drawback of the Findley equation is that determination of the material parameters is somewhat more complex and subjective than for the General Power-Law equation. The author suggests that further research be conducted to compare the Findley and General Power-Law equations for all types of polymeric geosynthetics.

4.2 Specific Recommendations for EPS-Block Geofoam

With specific regard to EPS-block geofoam, it would be useful to extend the results presented in this report to account for the following variables:

- Effect of material density on the Findley material parameters for small-strains under standard laboratory environmental (temperature and relative humidity) conditions. This essentially duplicates the conditions used by Magnan and Serratrice (1989) to develop parameters for the General Power-Law equation and would allow a thorough comparison of results for these two models. A comparison of the Findley and General Power-Law models for one EPS-block geofoam density as detailed in the Appendix of this report raises questions about the accuracy of the specific General Power-Law model parameters suggested by Magnan and Serratrice (1989).

- The effect of temperature, especially temperatures greater than de facto-standard laboratory conditions (+23°C (+73°F)), on the Findley parameters for a given EPS-block density under small-strain conditions. As discussed in Horvath (1995), there is preliminary evidence based on other, unpublished creep-test data obtained by BASF AG and reviewed in the past by the author that creep of EPS-block geofoam increases with increasing temperature and decreases with decreasing temperature. Up to this point in time, EPS-block geofoam usage has typically been in temperate to cold regions where the use of creep results obtained under warmer laboratory temperatures has simply meant conservative estimates of creep effects in practice. However, with the rapidly growing use of EPS-block geofoam as lightweight fill in warmer climates such as in Southeast Asia, the effect of temperature on EPS-block creep needs to be quantified. Therefore, long-term creep tests of EPS block:

    - with a benchmark density of 20 kg/m^3 (1.25 lb/ft^3),
    - at temperatures within the range of perhaps +35°C (+95°F) to +40°C (+104°F), and
    - applied stresses up to approximately 50 kPa (1000 lb/ft^2)

represent a high priority for future testing.

- The effect of a greater applied-stress range and concomitant large-strain conditions, such as encountered in compressible-inclusion applications, on the Findley material parameters for EPS block of a given density.

- Application to other commonly used polymeric geofoam materials such as elasticized EPS block, polystyrene porous block, and extruded polystyrene (XPS).

4.3 Acknowledgements

The author is grateful to BASF AG in Ludwigshafen, Germany for providing the creep-test data used in this study as well as other data and technical literature made available to the author since 1991. The author is particularly grateful to Herr Dipl.-Ing. Giselher Beinbrech, Herr Dipl.-Ing. Frieder Hohwiller, and Herr Dr. Krollman of BASF AG for generously sharing their insight and experience with EPS. However, the data interpretations and opinions expressed in this report are solely those of the author and do not necessarily reflect those of BASF or any of its employees.

Appendix

Introduction

Although not the primary focus of this report, it is of some interest to explore the differences in calculated results between the true Findley equation and the General Power-Law equation for EPS-block geofoam with a density of 20.3 kg/m^3 (1.27 lb/ft^3) which is the density of the test specimens used in the study reported herein. In particular, the coefficients for the General Power-Law equation that are used herein are those developed during the 1980s under the overall direction of the French federal government central road research laboratory, the Laboratoire Central Ponts et Chaussées (LCPC), and presented in Magnan and Serratrice (1989). The LCPC General Power-Law equation parameters are the only ones available for a relatively wide range of densities of EPS-block geofoam.

LCPC Version of the General Power-Law Equation

The standard form of the General Power-Law equation as stated in this report is

$$e = e_o + mt^n \tag{13}$$

However, to be consistent with the variables used in Magnan and Serratrice (1989), Equation 13 is rewritten here as

$$e = e_o + at^n \tag{A1}$$

The immediate-strain component, e_o, was assumed by LCPC to be linear elastic so Equation A1 can be rewritten as

$$e = \left(\frac{s}{E_{t_i}}\right) + at^n \tag{A2}$$

where E_{t_i} is the initial tangent Young's modulus of the EPS-block geofoam. Based on an extensive laboratory creep-test program conducted under the auspices of LCPC, recommendations for empirically evaluating the coefficients in Equation A2 were also given in Magnan and Serratrice (1989):

$$a = 0.00209\left(\frac{s}{s_y}\right)^{2.47} \tag{A3a}$$

$$n = -0.9\log_{10}\left[1 - \left(\frac{s}{s_y}\right)\right] \tag{A3b}$$

where:

s = the applied stress in kPa and

s_y = *yield stress* of the EPS in kPa (defined further subsequently).

Note that Equations A3 were formulated to produce a strain, *e*, in Equation A2 in decimal, not percent, form. Note also that Equation A3a is a corrected version of Equation 4.19 in Horvath (1995) which contained a typographical error.

The yield stress of EPS-block geofoam is a relatively new concept and parameter that is unfamiliar to most geofoam end users as well as geofoam manufacturers at this time and will be explained subsequently. However, at this point it is important to note that the yield stress of EPS-block geofoam should not be confused with the more-familiar foam material parameter of *compressive strength*.

To be consistent with Equations A3a and A3b which were developed empirically from a specific database, one must use the following empirical equations developed at the same time from the same database (Magnan and Serratrice 1989):

$$s_y = 6.41r - 35.2 \qquad \textbf{(A4a)}$$

$$E_{t_i} = 479r - 2875 \qquad \textbf{(A4b)}$$

where:

E_{t_i} has units of kPa,

r = EPS-block geofoam density in kg/m^3, and

s_y has units of kPa.

Combining Equations A2 and A3 produces

$$e = \left(\frac{s}{E_{t_i}}\right) + 0.00209\left(\frac{s}{s_y}\right)^{2.47} t^{\left\{-0.9\log_{10}\left[1-\left(\frac{s}{s_y}\right)\right]\right\}} \qquad \textbf{(A4)}$$

which, if Equations 4a and 4b are also included, produces the complete general form of the LCPC General Power-Law equation:

$$e = \left(\frac{s}{479r - 2875}\right) + 0.00209\left(\frac{s}{6.41r - 35.2}\right)^{2.47} t^{\left\{-0.9\log_{10}\left[1-\left(\frac{s}{6.41r-35.2}\right)\right]\right\}} \qquad \textbf{(A5)}$$

Again, note that in Equations A4 and A5 that the calculated strain, *e*, is expressed in decimal form, not as a percent.

Returning to the parameter of yield stress of EPS-block geofoam, it appears that the concept was first developed and proposed by the LCPC during the 1980s as part of their long-term study into the behavior of EPS-block geofoam as lightweight fill for road construction (Magnan and Serratrice 1989). The LCPC referred to this parameter not as the yield stress but as the *plastic stress*. However, the author prefers the term yield stress to plastic stress as this stress corresponds to a strain level of approximately 1.5% (Horvath 1995) and thus corresponds approximately to the beginning of the yield range of EPS-block geofoam. In any event, the primary utility of the yield-stress concept is to provide a normalization parameter for applied stress in developing stress-strain models for EPS-block geofoam which is exactly how LCPC used it as can be seen in Equation A4. It is of interest to note that Preber et al. (1994) used the same concept of a yield stress to develop an EPS-block geofoam stress-strain model for a somewhat different purpose although they did not reference the LCPC work in any way. Complete details for defining and determining the yield stress in practice, a discussion of its relationship to the compressive strength of EPS-block geofoam, as well as many other relevant details are given in Horvath (1995).

Comparison of Results

At this point it is of interest to compare results between the Findley equation and LCPC version of the General Power-Law equation using the test data in this report. The general form of the Findley equation is

$$e = e'_{o_F} \sinh\left(\frac{s}{s_{e_F}}\right) + m'_F \sinh\left(\frac{s}{s_{m_F}}\right) t^{n_F} \qquad \textbf{(6b)}$$

Using the BASF data evaluated for this study, the specific coefficients of Equation 6b were evaluated as discussed in Section 3.5 of this report to produce the following Findley equation specific for EPS-block geofoam with a density of 20.3 kg/m^3 (1.27 lb/ft^3):

$$\varepsilon = 0.011 \sinh\left(\frac{\sigma}{54.2}\right) + 0.000305 \sinh\left(\frac{\sigma}{33.0}\right) t^{0.20} \qquad \textbf{(15)}$$

This same EPS-block geofoam density was substituted into Equation A5 to produce the particular version of the LCPC version of the General Power-Law equation that is directly comparable to Equation 15:

$$\varepsilon = \left(\frac{\sigma}{6849}\right) + 0.00209\left(\frac{\sigma}{94.92}\right)^{2.47} t^{\left\{-0.9 \log_{10}\left[1-\left(\frac{\sigma}{94.92}\right)\right]\right\}} \qquad \textbf{(A6)}$$

Note that in both Equations 15 and A6 strain is expressed in decimal form.

The first item of interest to note qualitatively is that the exponents of *t* in Equations 15 and A6 (which are identical conceptually) are interpreted rather differently. Specifically, Findley assumed n_F to be stress-independent and the specific data investigated for this study which produced an average value for n_F = 0.20 support this. On the other hand, the LCPC results clearly indicate a stress dependence for *n*, with values for *n* ranging from 0.15 (for s = 30 kPa (630 lb/ft^2)) to 0.29 (for s = 50 kPa (1000 lb/ft^2)) for the stress-level range of interest here. Given that both parameters were evaluated for data within the linear-elastic range of EPS-block geofoam behavior, this deviation is significant both qualitatively (conceptually) and quantitatively.

Considering next a quantitative comparison of calculated results, Equations 15 and A6 were evaluated numerically. The results are shown in Figure A1 together with the actual creep-test data measured in the BASF tests used for this study. As can readily be seen, there are significant differences in the results. First of all, at small values of time where immediate strain dominates the calculated total strain the LCPC model predicts much smaller strains than measured for all three stress levels (and, of course, less than strains predicted using Findley material parameters based on those measured results). This is due to the fact that of all the sources for values of the initial tangent Young's modulus, E_{t_i}, assessed by the author

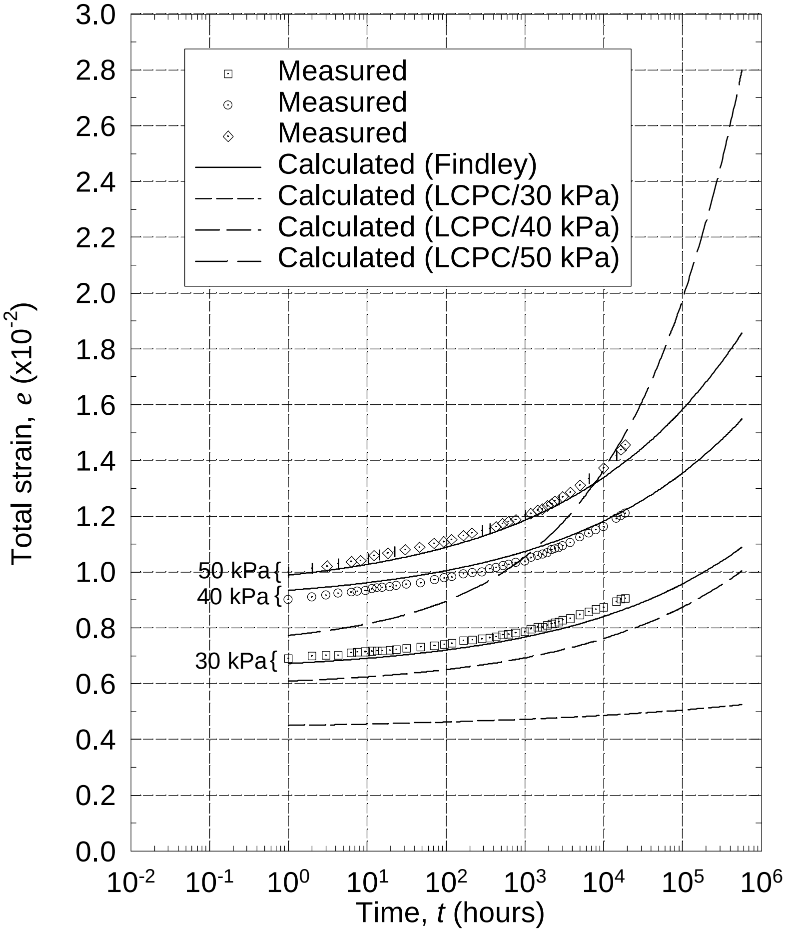

Figure A1. Comparison of measured versus calculated total strain v. $\log_{10}$ time

(Horvath 1995), the LCPC correlation (Equation A4b) consistently gives the largest values (E_{t_i} = 6850 kPa (143 kip/ft^2) for the EPS density considered in this study). On the other hand, as noted previously by happenstance the creep-test specimens used for this study had stiffnesses at the low end of the range of expected values for EPS-block geofoam of this density (an average E_{t_i} = 4900 kPa (100 kip/ft^2) in this case). Thus the LCPC correlation predicts a stiffness for the initial strain, e_o, that is approximately 40% greater than that actually measured in this study.

At larger values of time and for the largest applied stress (50 kPa (1000 lb/ft^2)), the LCPC model predicts significant creep strains of a magnitude that were neither measured in the actual test nor predicted by the Findley equation. A possible reason for this is that the LCPC model parameters may not have been developed from creep tests of sufficient duration. It is unclear from Magnan and Serratrice (1989) what the durations of loading were for the creep tests used to develop the LCPC model parameters. However, it is of interest to note that in Magnan and Serratrice (1989), correlations between actual creep-test data and calculated results were shown only for times less than or equal to 500 hours (approximately 21 days). This suggests that the creep tests performed by LCPC might not have been of a duration considered appropriate nowadays (10000 hours minimum and more if practical) and raises questions about the continued use of the LCPC version of the General Power-Law equation in practice. This also suggests that there is an imperative need to develop an accurate EPS-block geofoam creep model for use in practice.

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Notation

The following parameter notation is used throughout this report. Basic SI and imperial units for each parameter as typically used in practice are given in parentheses.

a	=	LCPC version of the General Power-Law equation material parameter (dimensionless)
E_{c_F}	=	Findley modulus for creep component of strain (kPa or kip/ft^2)
E_{o_F}	=	Findley modulus for initial loading (assumed elastic only, in kPa or kip/ft^2)
E_{t_i}	=	initial tangent Young's modulus (kPa or kip/ft^2)
E_{v_F}	=	Findley viscoelastic modulus (kPa or kip/ft^2)
m	=	material parameter (dimensionless)
$m¢_F$	=	Findley material parameter (dimensionless)
$m¢_H$	=	Horsley material parameter (dimensionless)
n	=	General Power-Law material parameter (dimensionless); also an LCPC General Power-Law equation material parameter (dimensionless)
n_F	=	Findley material parameter (dimensionless)
n_H	=	Horsley material parameter (dimensionless)
t	=	time after stress application (hours)
t_o	=	one hour; reference time used to normalize t
e	=	total strain at some time t after stress application (dimensionless)
e_c	=	creep strain at time t after stress application (dimensionless)
e_o	=	immediate strain upon stress application (dimensionless)
e'_{o_F}	=	Findley material parameter (dimensionless)
s	=	applied stress (kPa or lb/ft^2)
s_{e_F}	=	Findley material parameter (kPa or lb/ft^2)
s_{m_F}	=	Findley material parameter (kPa or lb/ft^2)
s_{m_H}	=	Horsley material parameter (kPa or lb/ft^2)