

New Structural Model for Multicomponent Pile Cross Sections under Axial Load

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Abstract: Piles composed of more than one material in their cross section have been used for more than 100 years. Originally this was limited to driven steel shell or pipe piles filled with portland-cement concrete. More recent developments include various types of drilled elements such as micropiles that consist of various combinations of steel shells, portland-cement grout, and steel reinforcing bars. The structural analysis or design of piles with multicomponent cross sections under axial load requires that the axial stress be apportioned to the various components. Traditionally this has been done using an approximate one-dimensional model that implies the components interact with each other only axially, not radially, and that there is no radial interaction with the ground around the pile. This note presents a new three-dimensional model that explicitly and rigorously considers not only the Poisson effects caused by axial load and the triaxial stress field that develops within and between components of a pile as a result but also how this stress field is affected by radial stresses in the adjacent ground. This new model is based on the theory of linear elasticity and yields a closed-form solution that can be either evaluated independently or incorporated within a more general analytical model for axial pile capacity. Examples of calculated results obtained using this new model are presented and suggest that Poisson effects are relatively small in magnitude so that the traditional one-dimensional model is adequate for routine use in most cases.

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Background

Traditional steel shell or pipe piles installed by impact driving and filled with fluid portland-cement concrete (PCC) after driving have been used for over 100 years (Chellis 1961). More recently, there has been increasingly greater use of drilled piles such as micropiles that consist of various combinations of steel shells, portland-cement grout (PCG) cores, and steel reinforcing bars placed at the center of the core. The rapidly growing use of micropiles in particular has provided impetus and incentive for taking a closer look at how the various pile components interact with each other structurally (FMSM 2003), a topic that has not received much attention over the years.

The structural analysis or design of any type of multicomponent pile under axial load requires that the axial stress caused by an assumed applied axial force be apportioned to the components comprising the pile cross section for the purposes of comparison to some allowable-stress value. Historically, this stress allocation has been done using a simple approximate one-dimensional (1D) analytical model that can be visualized as a system of independent axial springs oriented and acting in parallel with the longitudinal axis of the pile. Each material component of a pile cross section is assumed to be represented by its own spring. The

springs are usually assumed to be linear in behavior so that the equivalent spring stiffness of each component is simply AE per unit length, L , of pile where A and E are the cross-sectional area and Young's modulus respectively of each component. Note that where appropriate for cementitious components such as PCC or PCG, the time dependency of E should be taken into account (York et al. 1974). To develop a closed-form solution for this model, it is further assumed that the axial strains of the different components (i.e., displacements of the different "springs") are the same which is reasonable for any type of pile. It is then straightforward to develop a simple algebraic equation to apportion an applied force and calculate the respective axial stresses within each component. Relevant to the contents of this technical note, it is worth noting that this 1D model is conceptually and mathematically identical to assuming that each pile component exhibits linear-elastic behavior under uniaxial stress conditions which means that the axial stress in a pile component is dependent only on the axial strain and Young's modulus of that component.

There is a subtle implication with this 1D model that is perhaps not always appreciated in its practical application. The model and its results are theoretically correct only if the different components comprising the pile cross section do not interact with each other than having a common axial strain. Stated another way, the Poisson effects (radial and tangential stresses, radial displacements, and resulting radial interaction between and among the pile components and the outside ground) under axial load are neglected even though the different components are in intimate contact with each other, at least after initial construction and prior to load application. Therefore it should be recognized that this 1D model is not rigorously correct in practical applications and is always an approximation of reality because uniaxial stress conditions do not exist in the pile components.

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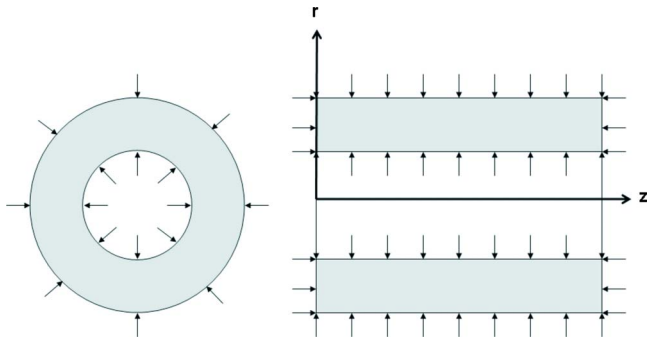


Fig. 1. Thick-wall cylinder in triaxial stress field

75 Scope

76 All indications are that this 1D model has produced acceptable
77 results in routine practice for over a century now. There is no
78 known pile failure that can be attributed to the use of this model.
79 Nevertheless, with the increasing use of new and different types
80 of piles consisting of multiple components and the renewed inter-
81 est in component interaction that has evolved with the increasing
82 use of micropiles in particular (FMSM 2003), it is of interest to
83 investigate the issue of three-dimensional (3D) effects between
84 and among pile components and the adjacent ground. With this in
85 mind, the scope of this technical note is to report on a new 3D
86 analytical model for this purpose that is theoretically rigorous yet
87 relatively easy to implement in practice. The analytical work to
88 date involving this new model has been limited to no more than
89 three components. However the basic theory and methodology
90 used for this new model are completely general so there is no
91 conceptual reason why the number of components could not be
92 increased without limit beyond 3. Suggestions as to how to ac-
93 complish this are also given in this note.

94 Proposed 3D Analytical Model

95 Concept and Theoretical Basis

96 The 3D model presented in this note is based on a theory of
97 linear-elasticity solution published in Poulos and Davis (1974)
98 (p. 297/Sec. 15.1). The particular solution used is called “thick-
99 wall cylinder in triaxial stress field” and is illustrated in Fig. 1
100 with the externally applied stresses assumed to be positive in sign
101 as shown. Note that Eq. 15.1f of this solution, which is for radial
102 displacement of the cylinder, contains a typographical error (the
103 positive sign that begins the second line of this equation should be
104 a negative sign) that was found to have significant impact on
105 calculated results for both the basic solution to the boundary-
106 value problem shown in Fig. 1 as well as the new 3D model
107 presented in this note. This error was only discovered after some
108 considerable effort during the validation phase (discussed subse-
109 quently) of this 3D model.

110 This new 3D model was obtained by using the basic solution
111 shown in Fig. 1 repeatedly to represent each component in the
112 pile cross section, effectively “nesting” the thick-walled cylinder
113 for each component within each other to create the desired
114 two- or three-component physical model. Thus this model could
115 easily be extended beyond three components simply by nesting
116 the basic solution additional times as desired. Note that the inner-
117 most component in both the two- and three-component versions

of this 3D model is solid cylindrical in geometry (it does not have
to be). The aforementioned thick-wall cylinder solution was used
for this solid cylinder as well by simply setting the inside radius
and applied-pressure terms to zero. As expected, the resulting
equations were those expected for a solid right-circular cylinder
based on other solutions found in Poulos and Davis (1974) and
Timoshenko and Goodier (1970).

Assumptions

The following additional equilibrium and compatibility assump-
tions were made concerning the components in the pile cross
section to develop this new 3D model:

- The sum of the axial forces carried by each component equals
the total axial force applied to the pile cross section;
- The longitudinal (axial or z axis) strains are the same in each
component;
- The radial (r axis) displacements are continuous and compat-
ible at each interface between components;
- The radial stresses are continuous and compatible at each in-
terface between components; and
- The radial stress on the outside of the shell, which represents
the horizontal stress in the ground adjacent to the pile after
installation, was assumed to be a known constant input vari-
able. However the model could be extended by leaving this
stress an unknown variable and linking its magnitude to radial
displacement of the exterior of the pile by using a theory such
as that developed by Kodikara (Kodikara and Moore 1993)
based on cylindrical cavity expansion.

Note that the assumption of radial displacement and stress conti-
nuity at component interfaces implies that no gap develops at an
interface even if tensile stresses develop. This was felt to be rea-
sonable as the combinations of cementitious and metallic materi-
als used in piles would be expected to have some tensile strength
due to interface adhesion. The assumption of radial displacement
and stress compatibility at component interfaces implies that the
magnitudes and signs of these parameters are the same in each
component on either side of the interface. This was felt to be
reasonable simply based on physical considerations.

These above-described assumptions collectively reduce the
number of unknown variables in the previously cited equations in
Poulos and Davis (1974) to the:

- Axial strain;
- Axial stress within each component;
- Radial stresses between components; and
- Radial displacements at the interfaces between components as
well as the outside of the pile.

Note that tangential stresses within components are initially un-
known as well and could be made part of the solution. However
these stresses were judged not to be of importance and were not
evaluated.

The unknown stress, strain, and displacement variables were
assembled into a system of simultaneous linear equations that
were solved using commercially available spreadsheet software to
illustrate the relative simplicity of the overall methodology. Other
solution alternatives such as purpose-written software using For-
tran or other programming language could obviously be used.
Separate software versions for two- and three-component piles,
named HEX2 and HEX3, respectively, were developed and used
for this study.

Table 1. Validation of Basic Linear-Elastic Solution Used as Basis for New 3D Model

Analytical method	Displacements, nm (in.)		
	Axial	Radial	
		Inside	Outside
Corrected closed-form solution in Poulos and Davis (1974)	2,199,510 (0.086595)	628,269 (0.024735)	96,076 (0.0037825)
Finite-element numerical solution using ANSYS	2,200,300 (0.086626)	626,872 (0.024680)	94,412 (0.0037170)

Note: (1) axial displacements defined as compression positive; (2) radial displacements defined as outward positive; and (3) signs of ANSYS results adjusted as necessary to be consistent with conventions given in Notes 1 and 2.

176 Validation

177 Introduction

178 This new 3D model was validated in two ways by investigating:

- 179 • The basic solution shown in Fig. 1; and
- 180 • The complete model as implemented in the aforementioned
- 181 HEX2 and HEX3 programs.

182 Basic Elastic Solution

183 The basic elastic solution presented in Poulos and Davis (1974) and shown in Fig. 1 was validated in two ways. First, the basic elements of this solution can also be found in other references. A solution in Timoshenko and Goodier (1970) (pp. 68–71) was the specific one used in this case. Note that this latter reference uses classic solid-mechanics notation of tension positive whereas all solutions in Poulos and Davis (1974) use the “geotechnical” notation of compression positive. It was actually through the material published in Timoshenko and Goodier (1970) that the aforementioned typographical error in Poulos and Davis (1974) was confirmed.

194 Second, an arbitrary problem was formulated and analyzed using the corrected equations of the closed-form solution in Poulos and Davis (1974) and compared to results from a finite-element (FE) continuum model solved numerically using ANSYS (Version 11.0). The following were the assumed problem parameters (refer to Fig. 1 for visualization):

- 200 • Cylinder height=127 mm (5.00 in.);
- 201 • Inside radius of cylinder=22,200 μm (0.875 in.);
- 202 • Outside radius of cylinder=757,400 μm (2.982 in);
- 203 • Axial stress (direction as shown in Fig. 1)=520 MPa (75 ksi);

- Radial stress on the inside of cylinder (direction as shown in Fig. 1)=690 MPa (100 ksi);
 - Radial stress on the outside of cylinder (direction as shown in Fig. 1)=345 MPa (50 ksi);
 - Young’s modulus of cylinder material=17 GPa (2,500 ksi); and
 - Poisson’s ratio of cylinder material=0.35.
- The calculated results are summarized in Table 1 and confirm the validity of the corrected solution in Poulos and Davis (1974) that was used as the basis for the new 3D model.

New 3D Model

As discussed subsequently, several test/example problems were analyzed using the HEX2 and HEX3 programs as appropriate to investigate the new 3D model. Four of these problems were also analyzed using FE continuum models solved numerically using ANSYS (Version 11.0) to provide validation of the complete 3D model. The ANSYS analyses used an axisymmetric model with a mesh consisting of 400 elements (20 radially by 20 vertically). In all four cases the results obtained using the new 3D model as implemented in the HEX2 and HEX3 softwares were essentially the same as those obtained using the ANSYS continuum model. This can be seen in Table 2 which presents specific results from one of these comparisons, specifically, the three-component example (referred to as FMSM Sample 3) with an externally applied radial stress. Further explanation of the composition of FMSM Sample 3 is given in the following sections but as can be seen in Table 2, the excellent comparison between the new 3D model and ANSYS results provides validation of the new 3D model.

Example Results

Introduction

As noted above, several test/example problems were analyzed to provide an overall assessment of typical results obtained using the new 3D model as well as to provide a comparison to results obtained using the traditional 1D model. Although the problems analyzed were not intended to be exhaustive of all reasonable combinations of pile materials and dimensions that might be encountered in practice they were chosen and felt to be representative. Particular emphasis was placed on analyzing micropiles in view of the fact that they have received particular attention in the last few years with regard to how components interact (FMSM 2003). Although the study of FMSM (2003) was for purposes completely unrelated to the present study and 3D model develop-

Table 2. Validation of New 3D Model (Micropile FMSM Sample 3 with External Stress)

Analytical method	Internal stresses, kPa (psi)					Axial strain (10^{-6})	Radial displacements, nm (in.)		
	Axial			Radial			Rebar-core interface	Core-shell interface	Shell-ground interface
	Rebar	Core	Shell	Rebar-core interface	Core-shell interface				
New 3D model	277,770 (40,257)	24,310 (3,523)	277,210 (40,175)	600 (87)	1,100 (159)	1,363	9,040 (0.000356)	33,810 (0.001331)	37,080 (0.001460)
ANSYS	277,770 (40,257)	24,310 (3,523)	277,125 (401,77)	607 (88)	1,100 (159)	1,363	9,040 (0.000356)	33,810 (0.001331)	37,080 (0.001460)

Note: (1) axial stresses defined as compression positive; (2) radial interface stresses defined as compression positive; (3) axial strain defined as compression positive; and (4) radial displacements defined as outward positive.

Table 3. Assumed Physical Properties of Pile Materials Analyzed for Present Study

Material	Elastic parameters	
	Young's modulus, MN/m ² (ksi)	Poisson's ratio
PCC	20,700 (3,000)	0.15
PCG	17,000 (2,500)	0.35
Steel	205,000 (29,700)	0.30

ment, two micropile sections from the report of FMSM (2003) were chosen for analysis as they were judged to be representative of what might be encountered in practice. In addition, the report of FMSM (2003) provided useful data concerning the relevant dimensions and engineering properties of micropile components.

251 Micropiles

Two micropile cross sections, one designated FMSM Sample 7 and comprised of two components (a steel shell with PCG core) and the other designated FMSM Sample 3 with three components (steel shell with PCG core and a steel rebar in the center of the core), were analyzed using data presented by FMSM (2003). In both cases the Schedule 40 steel shell had an outside diameter of 168 mm (6.625 in.) and an inside diameter of 152 mm (5.964 in.), and the PCG used had a nominal unconfined compressive strength $f'_c = 35$ MPa (5,000 psi). For Sample 3 the Grade 150 rebar was 44 mm (1.75 in.) in diameter. The assumed elastic properties of the micropile materials are given in Table 3. No adjustment was made to the PCG modulus developed from the data presented in FMSM (2003) to account for long-term behavior.

Analyses were performed using HEX2 and HEX3 as appropriate for each section both without and with an arbitrary externally applied radial stress to assess the effect of this variable on the calculated results. In addition, results for the traditional 1D model were also calculated for comparison (this 1D calculation was also incorporated into the HEX2 and HEX3 softwares). A simulated axial-compressive force of 2,000 kN (450 kips) was applied in all cases. The results are presented in Tables 4 and 5 for the two- and three-component sections respectively.

Driven Pile

A steel pipe pile filled with PCC was also analyzed to illustrate results for this traditional type of driven pile. The pile was assumed to have a 457-mm (18 in.) outside diameter with a wall thickness of 13 mm (0.5 in.). The assumed elastic parameters for the steel and PCC core are given in Table 3. The calculated results are given in Table 6 for a simulated axial-compressive force of 6,000 kN (1,350 kips). Note that the radial core-shell interface stresses are tensile in this case.

Summary

From the example problems presented in this note as well as additional analyses performed, several broad trends are observed:

- The calculated results are qualitatively very similar for both the micropile and driven-pile sections analyzed;
- The axial stresses within components obtained using the traditional approximate 1D model are very close in magnitude to those obtained using the new exact 3D model;
- The radial stresses at the interfaces between pile components

Table 4. Analysis Results for Two-Component Micropile (FMSM Sample 7)

Model	External radial stress, kPa (psf)	Internal stresses, kPa (psi)			Axial strain (10^{-6})	Radial displacements, nm (in.)	
		Axial		Radial		Core-shell interface	Shell-ground interface
		Core	Shell	Core-shell interface			
3D	0	30,260 (4,385)	345,770 (50,111)	1,260 (182)	1,703	42,900 (0.001689)	46,990 (0.001850)
	240 (5,000)	30,330 (4,394)	345,500 (50,072)	1,420 (206)	1,700	42,550 (0.001675)	46,630 (0.001836)
1D	Any	29,410 (4,262)	349,380 (50,635)	0	1,705	0	0

Note: Sign conventions defined in Table 2.

Table 5. Analysis Results for Three-Component Micropile (FMSM Sample 3)

Model	External radial stress, kPa (psf)	Internal stresses, kPa (psi)					Axial strain (10^{-6})	Radial displacements, nm (in.)		
		Axial			Radial			Rebar-core interface	Core-shell interface	Shell-ground interface
		Rebar	Core	Shell	Rebar-core interface	Core-shell interface				
3D	0	280,000 (40,579)	24,230 (3,512)	277,370 (40,199)	390 (57)	930 (135)	1,365	9,070 (0.000357)	34,140 (0.001344)	37,440 (0.001474)
	240 (5,000)	277,770 (40,257)	24,310 (3,523)	277,210 (40,175)	600 (87)	1,100 (159)	1,363	9,040 (0.000356)	33,810 (0.001331)	37,080 (0.001460)
1D	Any	279,980 (40,577)	23,570 (3,416)	279,980 (40,577)	0	0	1,366	0	0	0

Note: Sign conventions defined in Table 2.

Table 6. Analysis Results for Pipe Pile

Model	External radial stress, kPa (psf)	Internal stresses, kPa (psi)			Radial displacements, nm (in.)		
		Axial		Radial	Axial strain (10^{-6})	Core-shell interface	Shell-ground interface
		Core	Shell	Core-shell interface			
3D	0	18,170 (2,633)	188,840 (27,368)	-1,120 (-164)	894	38,460 (0.001514)	42,370 (0.001668)
	240 (5,000)	18,170 (2,633)	188,830 (27,367)	-959 (-139)	892	36,960 (0.001455)	40,870 (0.001609)
1D	Any	18,660 (2,705)	184,760 (26,777)	0	902	0	0

Note: Sign conventions defined in Table 2.

292 may be either compressive or tensile depending on the particu-
 293 lar problem. The nature of the pile components in general and
 294 their relative Poisson's ratios in particular are the key variables
 295 influencing these results. However, regardless of sign these
 296 stresses are always relatively small in magnitude compared to
 297 the axial stresses within components; and
 298 • The radial stress applied to the exterior of the pile has very
 299 little influence on the calculated results for the range in
 300 stresses studied.

301 Conclusions

302 The overall conclusion drawn from analyses performed using the
 303 new 3D analytical model presented in this note is that for all
 304 intents and purposes, the components of a pile cross section under
 305 axial load can be adequately analyzed structurally as having no
 306 significant radial interaction, either between and among the pile
 307 components or with the outside ground. Therefore the continued
 308 use of the traditional 1D model that ignores all radial effects
 309 would appear to be justified in routine practice. Nevertheless, in
 310 some cases it might be appropriate to consider 3D effects within a
 311 pile. This might occur:
 312 • With a pile of unusual or novel size, geometry, materials, or
 313 construction;
 314 • When some advanced analytical method for geotechnical pile
 315 capacity that considers radial displacements of the pile is used
 316 (e.g., Kodikara and Moore 1993); or
 317 • For more exact interpretation of strain-gauge measurements
 318 within an instrumented pile.
 319 In such cases the methodology presented in this note should prove
 320 useful.

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