

Practical Subgrade Model for Improved Soil-Structure Interaction Analysis: Model Development

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Abstract: This note outlines the theoretical development of a new subgrade model with inherent spring coupling called the Modified Kerr-Reissner (MK-R) model. The MK-R model is a novel hybrid that synergistically combines the advantages of mechanical and simplified-continuum subgrade models with the specific objective of producing a model that is straightforward to implement in routine practice.

DOI: 10.1061/(ASCE)GM.1943-5622.0000070

CE Database subject headings: Soil-structure interactions; Geotechnical models; Mat foundations; Raft foundations.

Author keywords: Soil-structure interaction; Models; Geotechnical models; Foundations; Mat foundations; Raft foundations.

Background

A subgrade model is a two-dimensional equation that approximates the three-dimensional load-displacement behavior of the ground. Such models have been a staple of research and practice for soil-structure interaction (SSI) analysis in foundation engineering that continues to the present even though more sophisticated continuum and material constitutive models have existed for decades. A review of the state of knowledge concerning subgrade models and the associated and oft-misunderstood SSI parameters of subgrade reaction and coefficient of subgrade reaction can be found in Horvath and Colasanti (2009) and Colasanti and Horvath (2010).

Winkler's Hypothesis, which is typically visualized as a system of independent axial springs, has been the subgrade model of choice since its postulation in the 19th century even though it has long been known to provide a poor approximation of actual subgrade behavior. The fundamental flaw of Winkler's Hypothesis can be interpreted as the lack of interaction or coupling between adjacent springs. This flaw has led to research dating back to at least circa 1940 to find an improved subgrade model that inherently incorporates spring coupling in its derivation and is thus fundamentally more accurate (Horvath 1979, 1988, 1989).

Although numerous subgrade models with inherent spring coupling have been developed none has seen widespread adoption and use to date. This is due to various pragmatic issues that hinder their implementation (Horvath and Colasanti 2009; Colasanti and Horvath 2010). This means the long-standing need for a practical subgrade model that is a theoretically sound improvement over

the inherently and seriously flawed Winkler's Hypothesis still exists.

Scope of Note

This note outlines the theoretical development of a new subgrade model, called the Modified Kerr-Reissner (MK-R) model, that uses a novel hybrid approach to produce what is believed to be the first subgrade model that inherently incorporates spring coupling and also meets the needs for use in routine practice. Companion papers demonstrate in detail how to implement the MK-R model in commercially available structural analysis software (Colasanti and Horvath 2010) and how to evaluate the model coefficients and parameters rationally using geotechnical engineering site-characterization methodologies (Horvath and Colasanti 2009).

Evolution of Subgrade Models

The evolution of subgrade models has been covered in detail previously (Horvath 1979, 1988, 1989) with overviews in Horvath and Colasanti (2009) and Colasanti and Horvath (2010). Of relevance is that there are two completely different and distinct theoretical approaches for developing subgrade models. The more commonly known mechanical approach uses arbitrary assemblages of mechanical elements such as axial springs, tensioned membranes, shear layers, and flexural layers. Winkler's Hypothesis, as visualized traditionally using the spring analogy, is the simplest mechanical model. The less-well-known simplified-continuum approach is based on making simplifying assumptions to the governing equations of a linear-elastic continuum.

As discussed in detail in Horvath and Colasanti (2009) and Colasanti and Horvath (2010), there are advantages and disadvantages to each approach that can be summarized as follows:

- It is easy to implement mechanical models in commercially available structural analysis software. However, the rational, problem-specific evaluation of the model parameters (spring stiffnesses, etc.) is not straightforward; and
- The parameters of simplified-continuum models (Young's

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Note. This manuscript was submitted on September 9, 2009; approved on June 10, 2010; published online on XXXX XX, XXXX. Discussion period open until July 1, 2011; separate discussions must be submitted for individual papers. This technical note is part of the *International Journal of Geomechanics*, Vol. 11, No. 1, February 1, 2011. ©ASCE, ISSN 1532-3641/2011/1-1-XXXX/\$25.00.

Table 1. Synthesis and Overall Hierarchy of Subgrade Models

Subgrade model				Derivative order of $p(x,y)$			Derivative order of $w(x,y)$		
			Hybrid	0	2	4	0	2	4
Mechanical	Simplified elastic continuum								
Winkler's Hypothesis	Winkler-type simplified continuum		(Trivial)	X			X		
Filonenko-Borodich	Pasternak-type simplified continuum		(None to date)	X			X	X	
Loof's hypothesis	Vlasov and Leont'ev (1 layer)								
Pasternak's hypothesis									
Modified Pasternak/Kerr	Reissner Simplified Continuum	Modified Kerr-Reissner		X	X		X	X	
Modified Kerr/Horvath-Colasanti	Vlasov and Leont'ev (2 layers)								
Haber-Schaim	(None to date)	(None to date)		X			X		X
(None to date)	(None to date)	(None to date)		X	X		X	X	X
Hetényi	(None to date)	(None to date)		X		X	X		X
Rhines	(None to date)	(None to date)		X	X	X	X	X	X

76 modulus, etc.) can be rationally evaluated but it is unclear how
 77 to model these parameters in commercial software.
 78 In fact it is the disadvantages of each approach that have kept
 79 more advanced subgrade models with inherent spring coupling
 80 from being used in practice to date. A way around this impasse is
 81 to draw on the best features of each approach to produce a hybrid
 82 model that has all of the advantages and none of the disadvan-
 83 tages of any single model. This novel concept is based on the fact
 84 that regardless of how a subgrade model was developed concep-
 85 tually it always has the same basic form of the equation governing
 86 the relationship between subgrade reaction, p , and surface dis-
 87 placement, W

$$88 \quad p + \nabla^2 p + \nabla^4 p + \dots = W + \nabla^2 W + \nabla^4 W + \dots \quad (1)$$

89 Note that the coefficients multiplying the p and W terms in Eq. (1)
 90 have been eliminated for clarity and that depending on the spe-
 91 cific model not all of the higher-derivative terms will appear.
 92 Using Eq. (1), a useful way to summarize and compare the vari-
 93 ous subgrade models identified to date in the literature is shown in
 94 Table 1 which is an updated version of those published previously
 95 in Horvath (1993, 2002). In this table, terms in Eq. (1) appear in
 96 the p - W equation defining the behavior of a model. Of relevance
 97 is that in several cases mechanical and simplified-continuum
 98 models of identical form have been developed. Only the coeffi-
 99 cients multiplying the terms appearing in Eq. (1) differ between
 100 models of identical form but different derivation approaches.
 101 Thus, it is possible to develop a hybrid model by equating the
 102 coefficients of two different (i.e., mechanical and simplified-
 103 continuum) models of equal form. The resulting hybrid model has
 104 all the advantages but none of the disadvantages of the individual
 105 mechanical and simplified-continuum models. This is because the
 106 hybrid model has coefficients defined using mechanical elements
 107 that can be modeled in a relatively straightforward manner using
 108 commercially-available structural analysis software. However,
 109 these otherwise abstract mechanical elements are correlated with
 110 the elastic parameters of the subgrade and the subgrade geometry,
 111 thus allowing evaluation of the mechanical elements to be made
 112 in a theoretically rigorous fashion that eliminates the guesswork
 113 and uncertainty that has always plagued the use of mechanical
 114 subgrade models in practice.

115 **MK-R Hybrid Subgrade Model**

116 **Introduction**

117 With reference to Table 1, the choice of models used to create a
 118 hybrid model was governed by the fact that the most advanced

simplified-continuum model developed to date is the Reissner 119
 Simplified Continuum (RSC) (Reissner 1958). The corresponding 120
 mechanical model used is based on the Modified Pasternak (later 121
 called Kerr) model (Rhines 1965; Kerr 1966; Jones and Xenopho- 122
 ntos 1976; Gazetas 1981; Kerr 1985; Kneifati 1985) although 123
 some key modifications described subsequently were made by the 124
 writers to improve its practicality of use. 125

Physical Components 126

The RSC model is shown in Fig. 1 together with the key simpli- 127
 fying assumptions made to produce the p - W equation for this 128
 model. The original solution developed by Reissner (1958) as- 129
 sumed a load applied directly to the subgrade surface [Fig. 1(a)]. 130
 The resulting equation is 131

$$132 \quad p - \left(\frac{GH^2}{12E}\right)\nabla^2 p = \left(\frac{E}{H}\right)W - \left(\frac{GH}{3}\right)\nabla^2 W \quad (2a)$$

with all terms defined in Fig. 1. Although a linear-elastic plate 133
 overlying the elastic layer [Fig. 1(b)] is not an essential compo- 134
 nent of the RSC such a plate is present in most applications of 135
 interest in practice. Its presence influences boundary conditions 136
 and the resulting equation governing the behavior of the RSC 137
 model. 138

The solution for a perfectly smooth plate-elastic layer interface 139
 is the same as Eq. (2a) (Horvath 1983). The other limiting case of 140

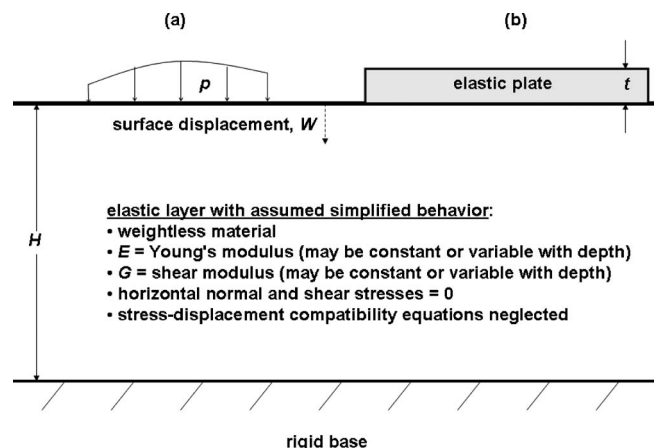


Fig. 1. Reissner Simplified Continuum

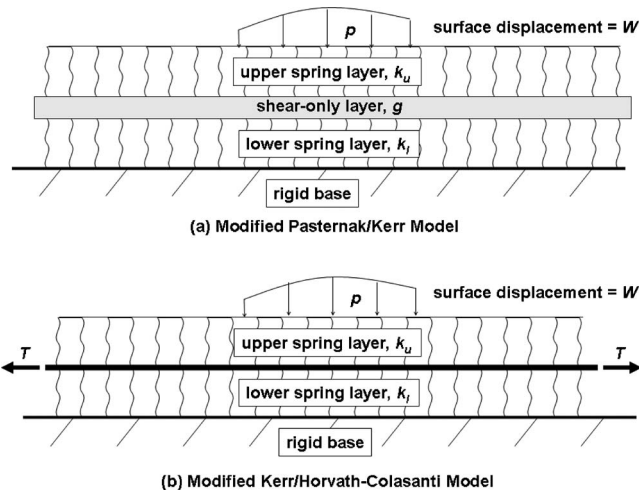


Fig. 2. Mechanical subgrade models considered in present study

a perfectly-rough plate-elastic layer interface was derived in Horvath (1979) and the governing equation for the subgrade component only is

$$p - \left(\frac{GH^2}{12E} \right) \nabla^2 p = \left(\frac{E}{H} \right) W - \left[\left(\frac{GH}{2} \right) \left(\frac{2}{3} - \frac{t}{2H} \right) \right] \nabla^2 W \quad (2b)$$

The Modified Pasternak/Kerr model is shown in Fig. 2(a) and consists of an upper layer of independent axial springs over an incompressible shear-only layer over a lower layer of independent axial springs. This model is essentially the Pasternak model with an additional upper layer of springs (Horvath 1979, 1988, 1989). Of relevance is that Pasternak simply assumed that spring coupling within the lower spring layer was provided by a vertical shearing resistance between adjacent springs without specifying any visualization or physical mechanism by which this would be accomplished. That this vertical shearing is provided by a shear-only layer was Kerr's (1961) nonunique physical visualization of how this might be accomplished.

Initial research by the writers indicated that modeling a shear-only layer in commercially available structural analysis software is somewhat difficult. Consequently, alternative visualizations were investigated for the interspring shearing assumed by Pasternak. Drawing on earlier work by Filonenko-Borodich (Horvath 1979, 1988, 1989), a perfectly flexible membrane under constant tension, T , was substituted by the writers for the shear-only layer to produce the same interspring shearing mathematically. As discussed in Colasanti and Horvath (2010), a tensioned membrane can be modeled readily.

The resulting mechanical model that substitutes a tensioned membrane for the shear-only layer is shown in Fig. 2(b) and is called the Modified Kerr/Horvath-Colasanti model. This model is formally identified for the first time in this note as a new mechanical subgrade model in its own right because it utilizes a different mechanical element to produce spring coupling compared to the original Modified Pasternak/Kerr model. The equation defining the behavior of the Modified Kerr/Horvath-Colasanti model is

$$p - \left(\frac{T}{k_u + k_l} \right) \nabla^2 p = \left(\frac{k_u k_l}{k_u + k_l} \right) W - \left(\frac{T k_u}{k_u + k_l} \right) \nabla^2 W \quad (3)$$

with all terms defined in Fig. 2(b).

Implementation in Practice

The system shown in Fig. 2(b) and expressed in Eq. (3) is what is physically modeled in commercially available structural analysis software. Details and guidelines for accomplishing this are given in Colasanti and Horvath (2010). The abstract mechanical elements are correlated to site-specific subgrade properties by equating the coefficients in Eq. (3) with the appropriate governing equation for the RSC [Eq. (2a) or Eq. (2b)]. These relationships are given in the Appendix.

Performance Verification

The accuracy of the MK-R model can be evaluated in two ways. The simpler and more fundamental way is to compare results for idealized problems involving one or more layers of linear-elastic material with relatively simple applied loads. The assumed correct baseline results for such problems can be obtained using closed-form solutions or explicit numerical analyses of continua.

The other type of evaluation involves case histories of actual structures where appropriate measurements have been made to allow comparison of measured values to those calculated using the MK-R model. However the use of geotechnical case histories is always complicated by the fact that determination of subgrade properties dominates the process and complicates the assessment of the accuracy of the analytical technique itself. Stated another way, even the most theoretically rigorous and correct analytical methodology can produce poor comparison with actual measurements if the subgrade properties input into the methodology are not appropriate.

In consideration of these factors, the assessment of the MK-R model presented in this note is limited to exact solutions of idealized elastic systems. Case-history comparisons are still desirable but this is left to a companion paper (Horvath and Colasanti 2009) where the subject of how to properly assess subgrade properties for use with the MK-R model is addressed in detail.

Two test problems are presented in this note. In each problem the baseline exact analysis was performed using a computer code named G183 (Colasanti 2008). This program can analyze plate-type structural elements bearing on a layered linear-elastic system using the solution developed by Schiffman (1962) and was extensively validated during its development by comparison to several published solutions.

Fig. 3 shows a single-layer problem. The calculated settlement of the elastic plate is shown in Fig. 4 along both the x - and y -axes of the plate. The good agreement between the baseline results and those obtained using the new MK-R model is apparent. Note also that the MK-R model results are superior to those obtained assuming a Winkler subgrade.

Fig. 5 shows a multilayer problem that was analyzed and involves the common problem of a uniformly loaded plate but with a complexity. This is because the MK-R model requires that any multilayer system be converted first into an equivalent isotropic, homogeneous, single-layer system in order to evaluate the MK-R model parameters. This is not a unique requirement as many analyses in foundation engineering require conversion of a multilayer system to an equivalent single-layer one. This can be done rationally using either a strain- or stress-weighted approach. A strain-weighted methodology presented by Fraser and Wardle (1976) was used for this study. The results are shown in Fig. 5.

Calculated plate settlements along the x -axis are shown in Fig. 6. Note that two baseline analyses were performed using the G183 program, one for the exact multilayer problem and one for

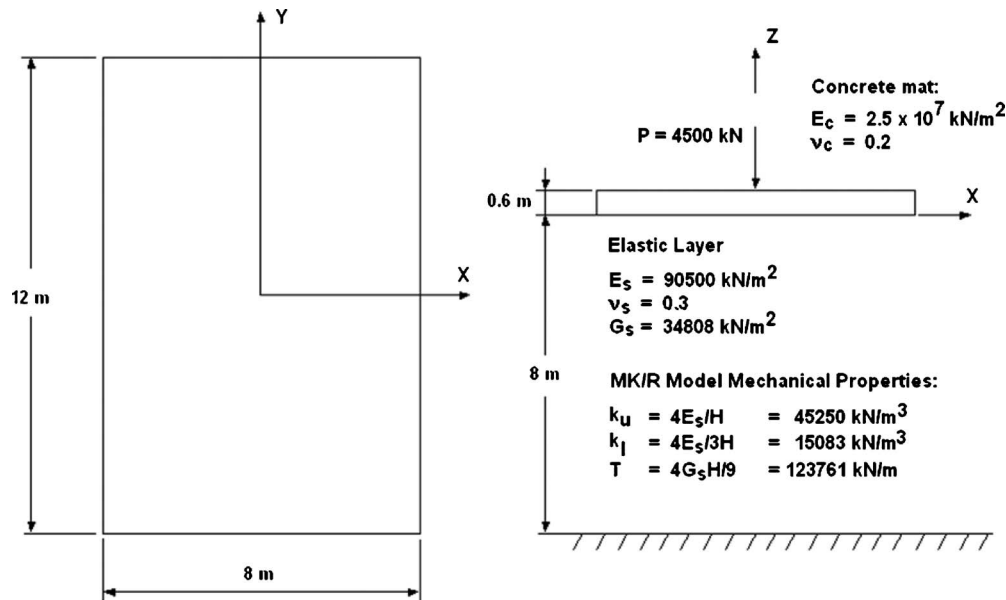


Fig. 3. Test Problem No. 1—problem details

238 the equivalent single layer approximation, to explore the sensitiv-
 239 ity of the exact results to this common approximation technique.
 240 As can be seen, the two results are very close. Next, the results
 241 obtained using the MK-R model are compared to the G183 results
 242 and it is clear that the MK-R model qualitatively captures the
 243 classical dishing pattern of settlement expected with a uniform
 244 load in addition to being quantitatively close to the baseline re-
 245 sults. The same cannot be said of the results using the Winkler
 246 model.

247 Conclusions

248 Although limited test cases for the MK-R model were presented
 249 in this note the results were similar in all key aspects to those
 250 obtained from much more extensive assessments done in the past
 251 for the RSC model (Horvath 1979, 1983) which is an integral part
 252 of the MK-R model and forms the theoretical basis for evaluating
 253 the MK-R model coefficients. Thus, the conclusion is that the new
 254 and novel MK-R hybrid subgrade model is the long-sought prac-

tical improvement to Winkler's Hypothesis for SSI analyses in 255
 routine practice. The companion papers on this topic (Colasanti 256
 and Horvath 2010 and especially the case histories presented in 257
 Horvath and Colasanti 2009) provide additional information in 258
 support of this conclusion. 259

Acknowledgments 260

The ANSYS (version 11.0) software and computer hardware on 261
 which it was installed that were used for all analyses involving 262
 the MK-R model were provided by URS Energy and Construction 263
 Division. This support is acknowledged with gratitude. The use of 264
 software trade names in this note is for necessary identification 265
 purposes only and does not constitute a recommendation or en- 266
 dorsement of that software by the writers or their employers. 267

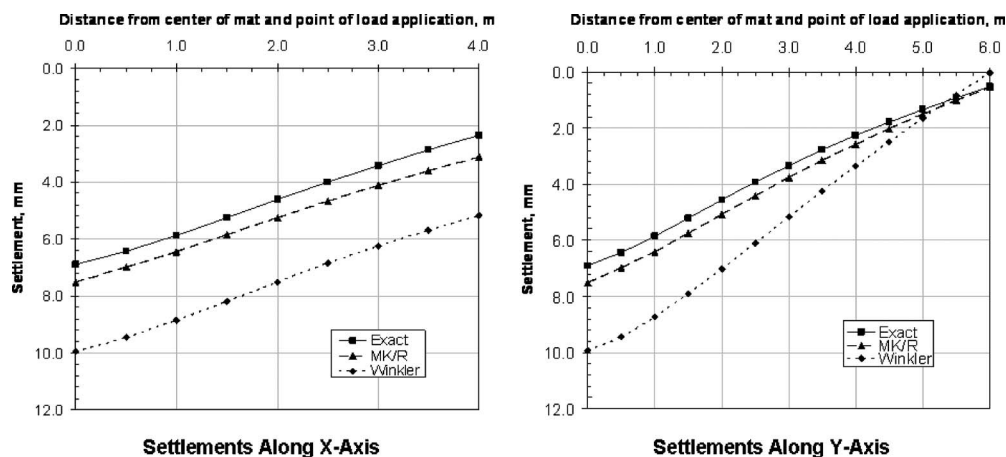


Fig. 4. Test Problem No. 1—calculated settlements

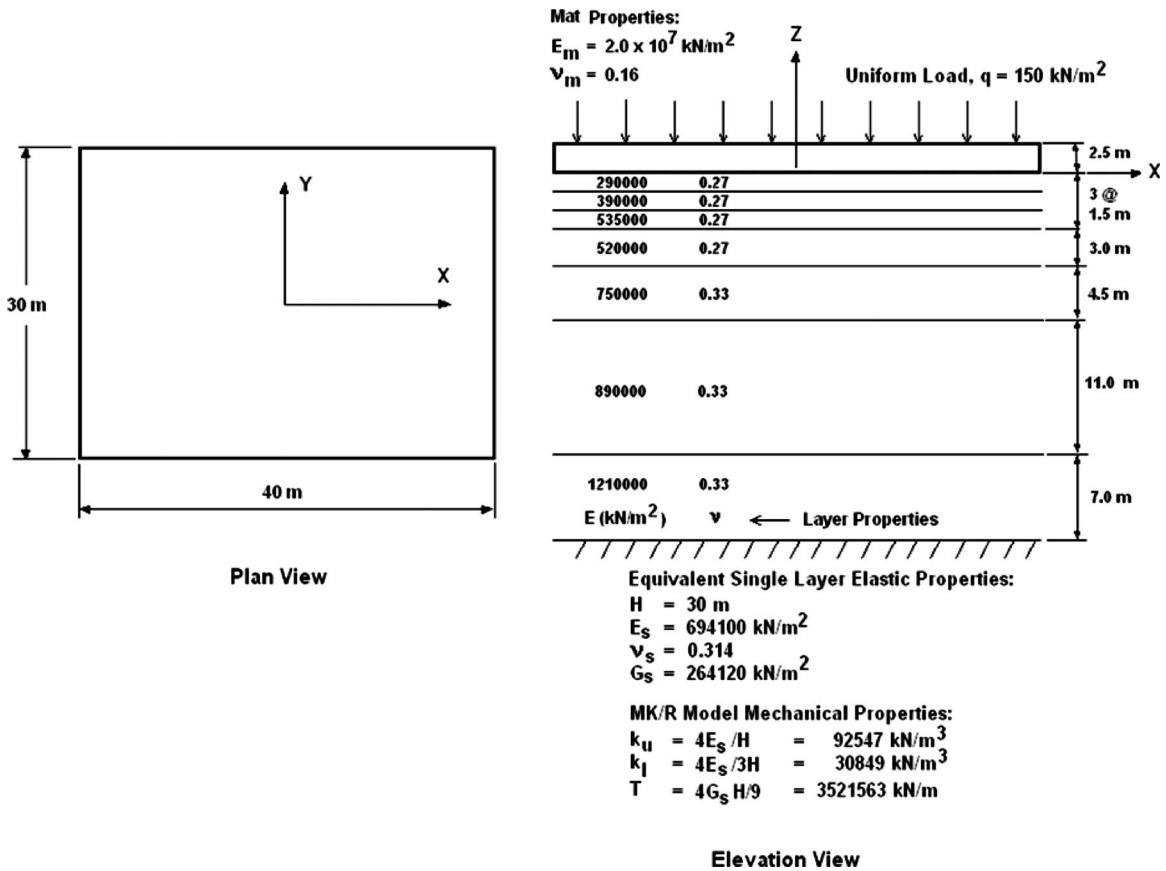


Fig. 5. Test Problem No. 2—problem details

Appendix: Parameters for MK-R Hybrid Subgrade Model 268
 269

No Plate or Perfectly-Smooth Plate-Subgrade Interface 270
 271

$$k_u = \frac{4E}{H} \quad (4a) \quad 272$$

$$k_l = \frac{4E}{3H} \quad (4b) \quad 273$$

$$T = \frac{4GH}{9} \quad (4c) \quad 274$$

Perfectly-Rough Plate-Subgrade Interface 275

$$k_u = \frac{E}{H} \left(\frac{4H-3t}{H} \right) \quad (5a) \quad 276$$

$$k_l = \frac{E}{3H} \left(\frac{4H-3t}{H-t} \right) \quad (5b) \quad 277$$

$$T = \frac{GH}{12} \left[\left(\frac{4H-3t}{H} \right) + \left(\frac{4H-3t}{3H-3t} \right) \right] \quad (5c) \quad 278$$

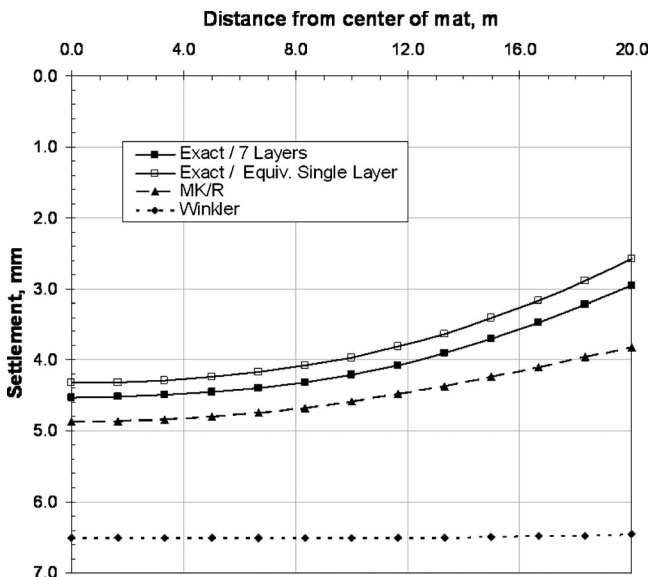


Fig. 6. Test Problem No. 2—calculated settlements

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