

Extended Veletsos-Younan Model for Geofoam Compressible Inclusions Behind Rigid, Non-Yielding Earth-Retaining Structures

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ABSTRACT: *Geofoam* is a type of *cellular geosynthetic* that can be used to reduce lateral earth pressures acting on rigid earth-retaining structures (RERSs) under a variety of soil and load conditions. One of the geosynthetically-functional ways in which geofoams do this is as a *compressible inclusion* to allow shear-strength mobilization of retained soil through *controlled yielding* which involves the well-known soil mechanics mechanism of *arching*. This paper describes how a mathematical model developed by Veletsos and Younan for a seismic-load application not involving geofoam was modified and extended to create a novel solution involving the use of geofoam as a compressible inclusion. An illustration of how this Extended Veletsos-Younan model can be applied in practice is also included using a case-history and full-scale, 1-g physical tests.

INTRODUCTION

Geofoam is the generic geosynthetic name for any closed-cell foam used in a geotechnical application (Horvath 1995). Contrary to popular, but incorrect, belief the term geofoam is not limited to one specific material or product. Geofoam is now recognized as a type of cellular geosynthetic along with geocombs and geocells.

Geofoams have many and varied functional applications. One with significant potential use worldwide and in a wide variety of ground and loading conditions is to reduce lateral earth pressures on earth-retaining structures (ERSs). A companion paper to this conference (Horvath 2008) addresses this overall geofoam application with an emphasis on seismic loading. This paper focuses on specific, innovative aspects of this broader subject.

BACKGROUND

Terminology

A term used in discussions related to ERSs is *yielding*. In this context, yielding is synonymous with horizontal (lateral) displacement.

Compressible Inclusions: Basic Concept

One of the more theoretically elegant yet practical functional uses of geof foam is as a *compressible inclusion* (Horvath 1995, 1996, 1998a, 2004, 2005). This is a *large-strain* geof foam function as the geof foam product used is designed to undergo relatively large compressive strains under design-load conditions.

The basic concept of a compressible inclusion is one that existed before geof foams were invented. Indeed the concept existed before modern soil mechanics evolved beginning in the 1920s. Simply stated, a compressible inclusion is any material that is intentionally the least-stiff component of a multi-material geotechnical system. As a result, the compressible inclusion readily compresses compared to the other system components under loads applied to the overall system. The net result is improved performance of the system, usually through load reduction (with concomitant safety increase and/or cost reduction) on one or more system components.

The earliest, classical application of the compressible-inclusion concept was using bales of hay or straw above the crown of underground conduits to reduce vertical loads on the conduits (Spangler and Handy 1982). This involved the soil-mechanics mechanism of *vertical arching* that is colloquially referred to as the *trap-door* effect (Terzaghi 1943). Arching can also develop in the horizontal direction (Handy 1985, Harrop-Williams 1989) which means that a compressible inclusion can be used to induce horizontal arching as well as vertical arching. Research has indicated that a durable, 'engineered' material such as geof foam is a superior material for compressible inclusions for both vertical and horizontal arching (Horvath 1995).

Compressible Inclusions: Geof foam Use to Reduce Lateral Earth Pressures on Earth-Retaining Structures

The basic, generic geof foam application of the compressible-inclusion concept with ERSs is known as the *Reduced-Earth-Pressure* (REP) concept and is illustrated in Figure 1. A relatively-thin (of the order of 300 mm (1 ft) thick or less) layer of geof foam that has been engineered and manufactured to have the desired stiffness is placed in a chimney-drain orientation between the back of the ERS and retained soil. In this application, the primary role of the geof foam product is to readily compress in its thickness (horizontal) dimension and allow horizontal displacement of the retained soil mass even if the ERS is rigid and non-yielding (e.g. a below-grade building wall, bridge abutment, or restrained retaining wall). As a result of this process, which is

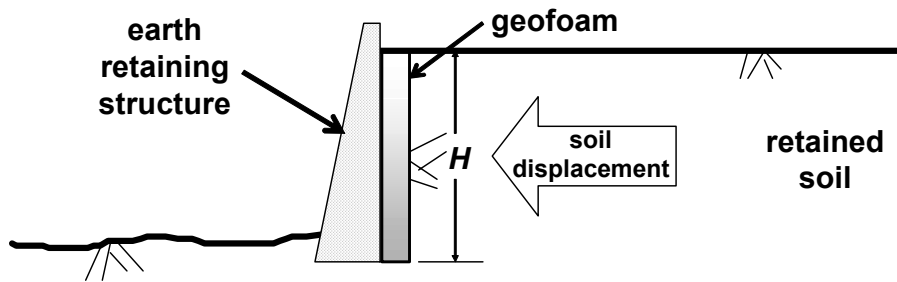


FIG. 1. Generic application of REP concept of compressible-inclusion function.

referred to as *controlled yielding*, the inherent shear strength of the retained soil is mobilized and the lateral earth pressure acting on the ERS decreases from the lateral pressure that would exist had the compressible inclusion not been there. Stated another way, a geofoam compressible inclusion used as shown in Figure 1 allows a retained soil mass to undergo the same horizontal displacement adjacent to a non-yielding RERS as it would have adjacent to a yielding one such as a free-standing retaining wall. This load-reduction benefit can be enjoyed for a variety of soil and loading conditions, as well as for both new construction as well as in the rehabilitation, renovation, or upgrading of existing structures. Of interest and relevance to this conference is use under seismic loading.

ANALYTICAL METHODS

Overview

The concept of using a geofoam compressible inclusion for lateral earth pressure reduction on RERSs has been proven by both research and actual use in practice going back at least to the 1980s. However most applications to date have involved gravity loads. Thus analytical methods for seismic loads are still an active, ongoing area of research and development. It is the primary purpose of this paper to discuss some of this ongoing research in order to encourage others to pursue research and advance the state of knowledge of this useful geotechnology.

Traditional Model

The traditional analytical model used to date for the REP concept illustrated in Figure 1 presumes use of this concept only with non-yielding RERSs that would normally be designed for the at-rest earth pressure state under both gravity and seismic loading. It is further assumed that the stiffness of the geofoam compressible inclusion is intentionally selected so as to allow horizontal displacement of the retained soil mass under the estimated earth loads (gravity or seismic) that is sufficient to mobilize the active earth pressure state within the retained soil. As discussed in Horvath (2004, 2005, 2008), the model concept used to date is that of two horizontally-oriented springs in series. One spring represents the compressive stress versus horizontal displacement in compression of the geofoam compressible inclusion and the other spring represents the compressive stress versus horizontal displacement in extension of the retained soil mass.

In most REP-concept applications, the primary design variable is the compressive stiffness of the compressible-inclusion 'spring'. This is defined as the Young's modulus of the compressible-inclusion material divided by the thickness (horizontal dimension in Figure 1) of the compressible-inclusion product installed. For the purpose of quantifying this stiffness it was found useful to define a new dimensionless parameter, λ , called the *normalized compressible inclusion stiffness* (Horvath 2000, 2004, 2005). This parameter is evaluated numerically using the following equation:

$$\lambda = \frac{E_{ci} \cdot H}{t_{ci} \cdot p_{atm}} \quad (1)$$

where E_{ci} = Young's modulus of the compressible-inclusion material; H = 'geotechnical' height of the RERS (see Figure 1); t_{ci} = thickness of the compressible inclusion; and p_{atm} = atmospheric pressure. Limiting values are $\lambda = 0$ (the 'perfectly compressible' case of horizontal displacement without restraint) and $\lambda = \infty$ (the 'perfectly rigid' case of no horizontal displacement, i.e. infinite restraint). Quantitatively, the smaller the value of λ the more compressible the inclusion is.

In the REP-concept application, the goal is to completely mobilize the inherent shear strength of the retained soil by allowing sufficient ground displacement so that the assumed active earth pressure state develops. This is presumed to occur after a certain magnitude of horizontal displacement, Δ_a , of the retained-soil mass (usually stated in the literature (e.g. Clough and Duncan 1991) as a dimensionless ratio, Δ_a/H). Therefore the stiffness of the compressible inclusion as defined by Equation 1 is matched to the soil stiffness necessary to mobilize the active state.

This simple-but-effective analytical model was first proposed by Partos and Kazaniwsky (1987) for gravity loading and used successfully on a project documented by them, with additional analysis and discussion of this case history by Murphy (1997). This model has gone through several evolutionary changes as discussed in detail in Horvath (2000). The most-useful form of the solution in practice is to solve for the required minimum thickness of the compressible inclusion:

$$t_{ci} = \frac{E_{ci} \cdot (\Delta_a / H)}{0.75 \cdot K_a \cdot \cos \delta \cdot \gamma_t} \quad (2)$$

where K_a = coefficient of active earth pressure evaluated numerically using either Coulomb or some 'exact' (e.g. log-spiral) theory; δ = friction angle between the retained soil and the geofoam compressible-inclusion material; and γ_t = total unit weight of the retained soil.

Equation 2 can be logically extended to seismic loading and is done so here as one new contribution of this paper:

$$t_{ci} = \frac{E_{ci} \cdot (\Delta_a / H)}{0.75 \cdot K_{ae} \cdot \cos \delta \cdot \gamma_t} \quad (3)$$

The only change from Equation 2 to Equation 3 is in the lateral-earth-pressure coefficient that appears in the denominator. K_{ae} in Equation 3 represents the seismic active earth pressure coefficient that could be evaluated, for example, from Mononobe-Okabe theory (Ebeling and Morisson 1992, Kramer 1996).

Alternative Model

While Equation 3 provides a simple, pragmatic model for use in routine practice it is of interest to explore alternative models that might capture the behavior of the

system more accurately yet still retain a simplicity of use that would be attractive in practice. The basis of one such model is the work of Veletsos and Younan (1994) who developed a mathematical model and corresponding numerical solution for a yielding RERS where yielding is restricted to the mode of rotation-about-base. They expressed the relative soil-wall-system stiffness using a dimensionless variable, d_θ , that was defined as follows:

$$d_\theta = \frac{G_{avg} \cdot H^2}{R_\theta} \quad (4)$$

where G_{avg} = average shear modulus of the retained soil (evaluation depends on the variation of shear modulus, G , with depth); H = geotechnical height of the RERS as before (see Figure 1); and R_θ = a rotational-spring stiffness that defines the rotational stiffness of the RERS on its foundation soil.

The author recognized subsequently that the applicability of the Veletsos-Younan model could be extended from its original boundary-value assumptions as their model could alternatively be visualized as a non-yielding RERS with a compressible inclusion placed between the RERS and retained soil as shown in Figure 1. In this case, even though the RERS cannot rotate about its base the retained soil can still go through the same rotational displacement as in the original Veletsos-Younan problem because of the compressible inclusion.

The postulation and detailed derivation of this extension of the Veletsos-Younan model was presented in Horvath (1997, 1998b). The final result (in a form simplified slightly from the original one published) is

$$d_\theta = \frac{2 \cdot G_{avg} \cdot t_{ci}}{H \cdot E_{ci}} \quad (5a)$$

which, when combined with Equation 1, can be expressed as

$$d_\theta = \frac{2 \cdot G_{avg}}{\lambda \cdot p_{atm}} \quad (5b)$$

where all terms have been defined previously.

Other Models

The use of geofom compressible inclusions to reduce lateral earth pressures on RERSs is receiving increasing attention from both practitioners and researchers. In addition to the traditional and Extended Veletsos-Younan models discussed in this paper, there are other models that have been developed in recent years by others. One such model can be found in Bathurst et al. (2007b). Consideration of this and other models is beyond the scope of this paper.

GEOFOAM MATERIAL AND PRODUCT SELECTION

As discussed in detail in Horvath (1995), there are numerous materials and products that have been used for geofoam applications in general. In particular, the concept of using geofoams as compressible inclusions has been investigated since at least the 1980s. Thus there is significant experience with determining the optimum geofoam materials and products to use for compressible-inclusion functional applications although both material and product refinement continues to the present.

At the present time and for the foreseeable future, expanded polystyrene (EPS) and materials related to it are the geofoam materials of choice for compressible inclusions. EPS is the ubiquitous white polymeric (plastic) foam used for beverage cups and cushion packaging. For the geofoam applications under discussion, EPS has been and can be used in various forms:

- *Block-molded* EPS is the simplest, most-generic product to use. All things being equal, the lower the EPS density the lower the material stiffness so typically the lowest-density EPS that is commercially available is used for compressible-inclusion applications. However, experience indicates that generic block-molded EPS is usually the least efficient choice from both technical and cost perspectives.
- A more-efficient form of block-molded EPS for compressible-inclusion applications is a product that has had a portion of its otherwise-solid cross-section removed so that there is less material per unit area in a direction parallel to the direction of desired displacement (horizontal in the case of Figure 1). A simple way in which this can be achieved is by creating a saw-tooth product (called *castellated* in product literature) by judicious factory cutting or by simply drilling holes (e.g. see Bathurst et al. 2007a, 2007b).
- The most-efficient form of EPS that has been found to date for compressible-inclusion applications is *resilient (elasticized)* EPS. This is generic low-density block-molded EPS that has been subjected to additional manufacturing steps to reduce its stiffness significantly below that of normal EPS, especially under relatively low-stress conditions (Horvath 1995).
- There is a geofoam material called *porous-polystyrene block* (PPB) that uses the same raw material (called *expandable polystyrene* or colloquially *bead* or *resin*) as EPS in the initial (*pre-expansion*) stage of its manufacture but with a completely different process used for its final stage of manufacture. Although PPB was developed for a completely different geofoam functional application (as a geosynthetic sheet drain for fluids) by happenstance PPB displays stiffness characteristics very similar to the aforementioned resilient-EPS. Thus PPB can be used for its compressible-inclusion functional characteristics. In fact, by pure happenstance it was the material used for the earliest known published case history involving geofoam compressible inclusions for lateral earth pressure reduction (Partos and Kazaniwsky 1987).

Regardless of the specific material or combination of materials used (e.g. PPB can be factory-laminated to resilient-EPS and a nonwoven geotextile to create a multifunctional geocomposite that acts as a compressible inclusion providing fluid drainage and thermal insulation as well), the products created from these materials for applications as shown in Figure 1 are typically factory-produced panels that are simply attached to the back of the RERS using a special adhesive to create the desired geometry.

CONCEPT VERIFICATION

Introduction

Published research and case-history applications to date involving geofam compressible inclusions have largely focused on gravity loading. However in recent years there has been increased interest in applications involving seismic loading so it is of interest to review available information in terms of the Extended Veletsos-Younan model discussed in this paper.

Numerical Modeling and Case-History Application

Inglis et al. (1996) presented a case history where low-density (13 kg/m^3 (0.81 lb/ft^3)) block-molded EPS was used around the below-grade portion of a building in Vancouver, British Columbia, Canada. A variety of numerical-modeling techniques were reportedly used during the design phase of the project. Although their published information is subject to some interpretation for the purpose of using Equation 1 to evaluate the relative stiffness of the compressible inclusion, calculations performed by the author resulted in λ values of the order of 50 to 100 for this project. Recall that the closer the value of λ is to zero the more effective the compressible inclusion should be at reducing lateral earth pressures compared to the baseline case of no compressible inclusion.

Using Equation 5b, this range in values of λ produce a relative soil-wall-system stiffness for the Extended Veletsos-Younan model, d_0 , of the order of 5 which suggests that the compressible inclusion in this case should be relatively effective at reducing lateral earth pressures. Note that for comparative purposes $d_0 = 0$ is the baseline case of no compressible inclusion and thus the larger the value of d_0 the more effective the compressible inclusion in terms of lateral-earth-pressure reduction.

Inglis et al. indicated that their various numerical analyses suggested that the geofam compressible inclusion reduced design loads by approximately 50% compared to the baseline condition of no compressible inclusion. This order-of-magnitude reduction is consistent with what one would expect for a $d_0 = 5$ based on figures published in Veletsos and Younan (1994).

Large-Scale Model Testing

Bathurst et al. (2007a) presented the results of 1-g laboratory testing involving relatively large-scale shake-table testing of a physical model that consisted of a 1-

metre-high rigid, non-yielding wall with a geofoam compressible inclusion. They tested three different compressible-inclusion materials and products:

- generic normal, moderate-density (16 kg/m^3 (1.0 lb/ft^3)), block-molded EPS;
- resilient-EPS (a proprietary commercial product); and
- generic normal, low-density (12 kg/m^3 (0.75 lb/ft^3)), block-molded EPS that had been modified to have one-half of its cross-sectional area removed by coring holes.

Using Equation 1, the author calculated that the relative stiffnesses of the compressible inclusions, λ , were 317, 86, and 26 respectively. Note that these values cover a larger range than the $\lambda \approx 50$ to 100 for the Inglis et al. (1996) case history discussed previously. Again recall that the smaller the value of λ the more effective the compressible inclusion should be at reducing lateral earth pressures compared to the baseline case of no compressible inclusion ($\lambda = 0$ is the limiting case of zero lateral earth pressure or a 'perfect' compressible inclusion).

The author's calculations using Equation 5b of the relative soil-wall-system stiffness, d_θ , for the Bathurst et al. tests were complicated by the fact that evaluating G_{avg} using the data provided by Bathurst et al. (2007a) was inconclusive. However using additional data in a companion paper (Bathurst et al. 2007b) it was possible to estimate values for $d_\theta = 0.4, 2,$ and 5 respectively for the three types of compressible inclusion tested ($\lambda = 317, 86,$ and 26 respectively). This is a relatively large range in relative soil-wall stiffness. Again, note that the larger the value of d_θ the more effective the compressible inclusion should be at reducing lateral earth pressures ($d_\theta = 0$ represents the baseline case of no compressible inclusion and no lateral-earth-pressure reduction).

Measurements of wall forces reported by Bathurst et al. (2007a) suggest that the compressible inclusions produced reductions in lateral earth pressures in the range between approximately 15% and 35% relative to the baseline case of no compressible inclusion. The relative magnitude of load reduction was consistent with what would be expected based on the relative values of λ and d_θ , i.e. the least-efficient compressible inclusion (a combination of the largest value of λ and smallest value of d_θ) resulted in the smallest load reduction, etc. The range of load reductions observed were also consistent with what one would expect based on figures published in Veletsos and Younan (1994).

CONCEPT EXTENSION

It is of interest to note that the REP concept shown in Figure 1 can be logically extended by including multiple layers of geosynthetic tensile reinforcement (geotextiles, geogrids, metallic elements) within the retained soil (Horvath 1995, 1996, 1998a, 2004, 2005, 2008). This is referred to as the *Zero-Earth-Pressure* (ZEP) concept because the lateral earth pressures acting on the ERS can be reduced to effectively zero by using appropriate combinations of compressible-inclusion and

reinforcement stiffnesses. This reduction accrues from the mobilization of resistance within the tensile reinforcement in the classic geotechnical mechanism of *mechanically stabilized earth* (MSE).

The ZEP concept is much less advanced than the REP concept in terms of model development, physical testing, and use in practice. However all research to date supports the validity of the basic ZEP concept and it is possible that the Extended Veletsos-Younan model discussed in this paper could be utilized if the horizontal stiffness of the reinforced retained soil mass were properly quantified, perhaps using elastic theory (e.g. Harrison and Gerrard 1972) as noted in Horvath (2008).

CONCLUSIONS

The use of geofoam compressible inclusions consisting of EPS and/or related materials in what is referred to as the REP concept is a proven geotechnology to reduce lateral earth pressures on RERSs under both gravity and seismic loading. The Extended Veletsos-Younan model shows promise as an analytical tool for analyzing and designing geofoam compressible inclusions under seismic loading and deserves further research and verification for this purpose.

One of the things to keep in mind when using compressible inclusions in a REP application is that the reduction in lateral earth pressures is not without limit. In any given application for both gravity and seismic loading there is always a stiffness of compressible inclusion (as characterized by the thickness of the inclusion) beyond which using a thicker inclusion with less stiffness would not have any technical benefit, at least in terms of lateral-earth-pressure reduction (it might in terms of other geofoam functions such as thermal insulation). This was observed in the numerical modeling performed by Inglis et al. (1996) and, importantly, is a feature captured by the Extended Veletsos-Younan model as well as the earlier, simpler traditional model that was also discussed in this paper.

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