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New Hybrid Subgrade Model for Soil-Structure Interaction Analysis: Foundation and Geosynthetics Applications

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ABSTRACT

The Modified Kerr-Reissner Model is a new advanced subgrade model developed using a unique hybrid derivational approach. It synergistically combines the best features but none of the flaws of the traditional mechanical and elastic-continuum approaches when used alone for developing subgrade models. The result is a model that is straightforward to implement in commercially available structural analysis software and has parameters defining its mathematical behavior that can be evaluated logically on a project-specific basis using conventional geotechnical engineering methodologies. Most importantly, because this new model inherently incorporates vertical shearing (referred to as *spring coupling* in subgrade modeling) into its mathematical formulation it eliminates all of the well-known theoretical shortcomings as well as other uncertainties of Winkler's Hypothesis, the predominant subgrade model used to date. This new model is thus of immediate use to practitioners and researchers alike for application in a wide range of soil-structure interaction problems. Examples are given for foundation engineering as well as with planar geosynthetics used for tensile reinforcement with vertical loading.

BACKGROUND

Subgrade models are mathematical expressions in two-dimensional space that approximate the load-displacement behavior of three-dimensional earth masses. They are not complete constitutive models but are, by intent and design, limited to capturing the key behavioral aspects of specific *soil-structure interaction* (SSI) applications involving relatively flexible structural elements in contact with or embedded within the ground. For such structural elements, correct problem solution requires not only satisfying force and moment equilibrium but also compatibility of the displacement and deformation pattern between structure and ground along their mutual contact surface(s).

Although subgrade models are inherently approximations of reality, they have been an important component of both geotechnical engineering practice and research for over 200 years. This is still true at present and into the foreseeable future despite the availability of more rigorous continuum and constitutive models. The reason is the relative simplicity, adaptability, and analytical efficiency of subgrade models in a wide variety of SSI applications. Historically, most applications have involved structural foundation elements such as mats (rafts), slabs-on-grade, 'rigid' portland-cement concrete pavements, and laterally loaded deep foundations. For the last 20-plus years this usage has been extended to include geosynthetics, primarily sheet-like planar products used as tensile reinforcement under vertical loading. A detailed overview and discussion of subgrade models including cited references giving examples of their many applications, including with geosynthetics, can be found in Horvath (2002).

Winkler's Hypothesis is the oldest subgrade model and remains the most popular one in use but only because it is so well known and easy to implement in practice. However, it is not only the least accurate subgrade model but also does a very poor job of modeling actual subgrade behavior, primarily because it does not inherently replicate the shearing capability of real subgrade materials. This significant behavioral shortcoming is usually referred to as a lack of *spring coupling*. This term derives from the fact that the most common and enduring visualization of the mathematically-abstract Winkler Hypothesis is that of a layer of independent axial springs supported on a rigid base. The inherent shearing resistance that is such a key behavioral aspect of real subgrade materials can be visualized as coupling or linkage between adjacent springs. It is this coupling or linkage that is mathematically lacking in Winkler's Hypothesis. The implications and consequences of this inherent, fatal lack of spring coupling in Winkler's Hypothesis are discussed in detail in Colasanti and Horvath (2010).

MODIFIED KERR-REISSNER HYBRID SUBGRADE MODEL

Subgrade Model Development

Research into subgrade models more advanced than Winkler's Hypothesis, all of which inherently include spring coupling in their mathematical formulation, has been conducted since at least the first half of the 20th century (Horvath 1979, 1989). There are two completely different conceptual approaches for developing subgrade models (Horvath 1979, 1989; Horvath and Colasanti 2011). The one more familiar to civil engineers is the *mechanical* approach that consists of building models by starting with a rigid base and adding various combinations of mechanical elements such as axial springs, constant-tension membranes, shear-only layers, and flexure-only plates (the latter three elements are used to create spring coupling). It has been show that, in the limit, an infinite number of such mechanical elements approaches the behavior of a linear-elastic continuum (Horvath 1979, 1989). Winkler's Hypothesis as typically visualized as only a layer of independent axial springs is thus the simplest mechanical model than can be conceived and the only one without inherent spring coupling.

The other derivational approach is the *simplified elastic continuum* concept that involves making arbitrary simplifying assumptions concerning the stress and displacement terms in the equilibrium, constitutive, and compatibility equations defining the behavior of a linear-elastic layer of finite thickness resting on a rigid base. The remaining simplified equations are then solved algebraically to create a mathematical equation defining the force-displacement behavior of the simplified elastic mass. Note that whereas with mechanical models one starts with nothing then increases the model complexity (and accuracy) with the simplified continuum approach one starts with the exact problem then decreases the model complexity (and accuracy). So in essence these two approaches work from different directions to achieve a model that strikes a balance between theoretical accuracy and complexity (which ultimately translates into ease of use).

Each of these derivational approaches has its pluses and minuses. Mechanical models are easy to visualize, model, and solve using commercially available structural analysis software but present a challenge with regard to problem-specific evaluation of the constituent mechanical elements. For example, the classical difficulty of determining the 'soil spring constant' for Winkler's Hypothesis is well known. If more mechanical elements are added to improve the accuracy of the model the difficulty in parameter evaluation rapidly becomes practically impossible. On the other hand, simplified continuum models have parameters that are straightforward to evaluate but implementing such models in existing software is problematic. Collectively, it is these pragmatic implementation issues that have prevented the widespread acceptance and use in practice of subgrade models more advanced (and thus more theoretically correct) than Winkler's and is the primary reason Winkler's Hypothesis remains predominant in both practice and research to the present.

Components

What makes the Modified Kerr-Reissner (MK-R) Model unique among advanced subgrade models and thus gives it its unique advantage of ease of use compared to all other advanced models is that it was developed using a hybrid derivational approach (Horvath and Colasanti 2011). As such, it makes synergistic use of both the mechanical and simplified continuum approaches for subgrade model development so that it has all of the pluses but none of the minuses of a pure mechanical or pure simplified continuum model.

The MK-R Model is essentially the Modified Kerr (a.k.a. Horvath-Colasanti) mechanical model and Reissner's Simplified Continuum (RSC) model combined synergistically into one model by selecting certain elements of each model as follows.

The Modified Kerr Model, which is shown in Fig. 1 (note that the spatial orientation of the model is completely arbitrary), is what is physically modeled in commercially available structural analysis software. Specific, detailed guidelines on how to do this for *ANSYS* (Version 11) are given in Colasanti and Horvath (2010).

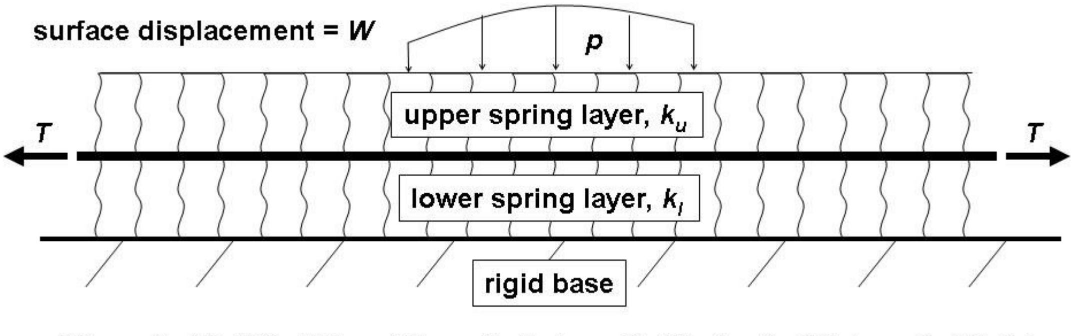


Figure 1. Modified Kerr (Horvath-Colasanti) Mechanical Subgrade Model.

This model consists of two layers of independent axial springs with stiffnesses k_u and k_l sandwiching a perfectly flexible membrane under constant tension, T. Note that it is the differential vertical displacement of the tensioned membrane that occurs as a consequence of vertical displacement of the lower spring layer that provides the vertical shearing force between adjacent springs that produces the desired spring-coupling effects. The equation defining the vertical force-displacement behavior of this model is

$$p-\left(\frac{T}{k_u+k_l}\right)\nabla^2 p=\left(\frac{k_u k_l}{k_u+k_l}\right)W-\left(\frac{T k_u}{k_u+k_l}\right)\nabla^2 W \qquad \textbf{(1)}$$

with all terms defined in Fig. 1 (Horvath and Colasanti 2011).

Note that the Modified Kerr Model if used by itself displays all of the classic advantages and disadvantages of a mechanically-derived subgrade model. While the model itself is easy to visualize physically and model using structural analysis software it is neither obvious nor intuitive how the values of the model coefficients (k_u, k_l, and T) are to be evaluated in any practical application. It is this inherent difficulty with all types of mechanical subgrade models, including Winkler's Hypothesis with just a single layer of springs, that has been an ongoing issue with their use in practice and has made the use of more-advanced mechanical subgrade models such as the Modified Kerr Model essentially impossible from a practical perspective.

Considering next the RSC Model shown in Fig. 2 (again note that the spatial orientation of the model is completely arbitrary), as discussed subsequently this model is what is used to visualize and characterize the MK-R Model from a geotechnical perspective in terms of physical conditions and soil properties in a site-specific application. The concept of developing subgrade models based on a simplified elastic continuum was first suggested by Reissner (1958). The specific model developed by Reissner for the assumptions shown in Fig. 2 yields the following equation defining the vertical load-displacement behavior of the model:

$$p-\left(\frac{GH^2}{12E}\right)\nabla^2 p=\left(\frac{E}{H}\right)W-\left(\frac{GH}{3}\right)\nabla^2 W \qquad \textbf{(2)}$$

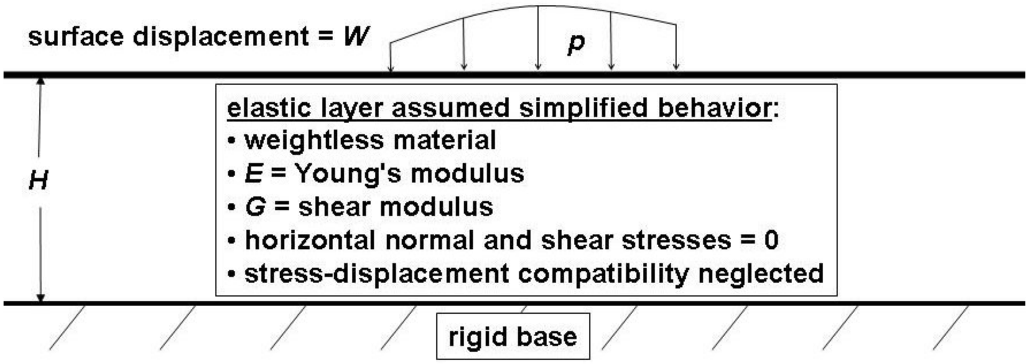


Figure 2. Reissner Simplified Continuum Subgrade Model.

with all terms defined in Fig. 2 (Horvath 1979, 1983a). This simplified continuum approach to subgrade modeling has since been extended to produce other models, including showing that an equation identical to Winkler's Hypothesis can be derived in this manner (Horvath 1979, 1983b).

Note that the RSC Model if used by itself displays all of the classic advantages and disadvantages of a simplified continuum subgrade model. While the model itself is easy to visualize physically and values of the model coefficients (*E*, *G*, and *H*) can be evaluated in a rational, straightforward manner in any practical application (Horvath and Colasanti 2009) there are significant issues with implementing this model in commercially available software. All studies of the RSC Model to date have always involved purpose-written computer software (Horvath 1979, 1983a). It is this inherent implementation difficulty with simplified continuum subgrade models that has hampered their use in practice for over a half-century.

Parameter Assessment

The innovative, unique aspect and real benefit of the MK-R Model comes from the synergistic way in which the Modified Kerr and RSC models are combined to form the MK-R Model and, in the process, eliminate the above-described negative aspects of each model if considered independently. That these two models based on completely different derivational approaches can be equated is theoretically correct because these models are of the same order of accuracy (Horvath and Colasanti 2011) and have the same form of the partial differential equations defining their behavior as is clearly evident by comparing Eqs. 1 and 2.

As noted previously, the physical visualization shown in Fig. 1 is what is used for structural analysis purposes for the MK-R Model as this assemblage of mechanical elements is what is implemented in commercially available structural analysis software (Colasanti and Horvath 2010). To eliminate the problem of how to rationally evaluate the mechanical elements shown in Fig. 1 on a problem-specific basis Fig. 2 is used to visualize the MK-R Model for geotechnical analysis purposes. The coefficients appearing in the equations defining the behavior of the Modified Kerr Model (Eq. 1) and RSC models are equated so that the abstract mechanical elements (spring stiffnesses k_u and k_l and membrane tension *T* shown in Fig. 1) are defined explicitly in terms of the physical conditions shown in Fig. 2 which can be related to the actual thickness and stiffness of the subgrade soils at a project site (a suggested procedure is summarized in Colasanti and Horvath (2010) and outlined in detail in Horvath and Colasanti (2009)):

$$k_u = \frac{4E}{H} \tag{3a}$$

$$k_l = \frac{4E}{3H} \tag{3b}$$

$$T = \frac{4GH}{9}. \quad \textbf{(3c)}$$

Note that if there is a structural element (mat, slab, etc.) in contact with the subgrade the surface stress p shown in Figs. 1 and 2 becomes the *subgrade reaction* (structure-subgrade contact or bearing stress). The relationships in Eqs. 3a-c are still valid for this case but only for the assumption of a perfectly-smooth structure-subgrade interface. The relationships for a perfectly-rough interface are slightly different:

$$k_u = \frac{E}{H}\left(\frac{4H-3t}{H}\right) \quad \textbf{(4a)}$$

$$k_l = \frac{E}{3H}\left(\frac{4H-3t}{H-t}\right) \quad \textbf{(4b)}$$

$$T = \frac{GH}{12}\left[\left(\frac{4H-3t}{H}\right)+\left(\frac{4H-3t}{3H-3t}\right)\right] \quad \textbf{(4c)}$$

where t = thickness of the structural element in contact with the subgrade.

Example Applications

Foundations

The RSC Model that is a key component of the MK-R Model has been studied extensively going back to the 1970s for applications involving plate-type shallow foundations such as mats (raft), slabs-on-grade, and 'rigid' pavements (Horvath 1979, 1983a). Fig. 3 shows a simple problem of this type taken from Horvath and Colasanti (2011) with calculated settlements shown in Fig. 4. The results shown are typical of those from various studies (Horvath 1979, Horvath and Colasanti 2011) and indicate that the MK-R Model results both compare favorably with those from various 'exact' solutions and are noticeably and consistently better than those obtained using Winkler's Hypothesis.

Geosynthetics

Of particular relevance and interest to this conference is that beginning in the mid-1990s there was a surge of research and concomitant publication on the subject of treating sheets or layers of planar geosynthetic tensile reinforcement of a soil subgrade subjected to vertical loading as a SSI problem using subgrade models as opposed to using traditional limit-equilibrium analytical methodologies (Horvath 2002). Consequently, to illustrate the use of subgrade models in general and the MK-R Model in particular for performing SSI-based analysis of problems involving sheet-type geosynthetic tensile reinforcement, the simple problem illustrated in Fig. 5 was analyzed using *ANSYS* (Version 11.0) with three different models for the subgrade:

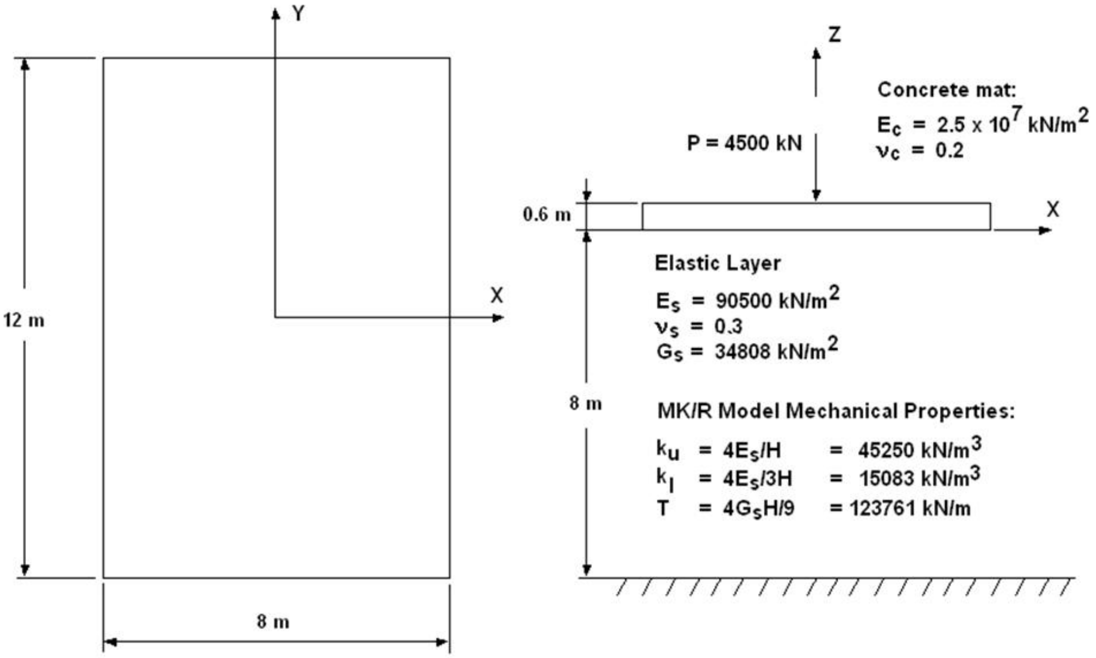
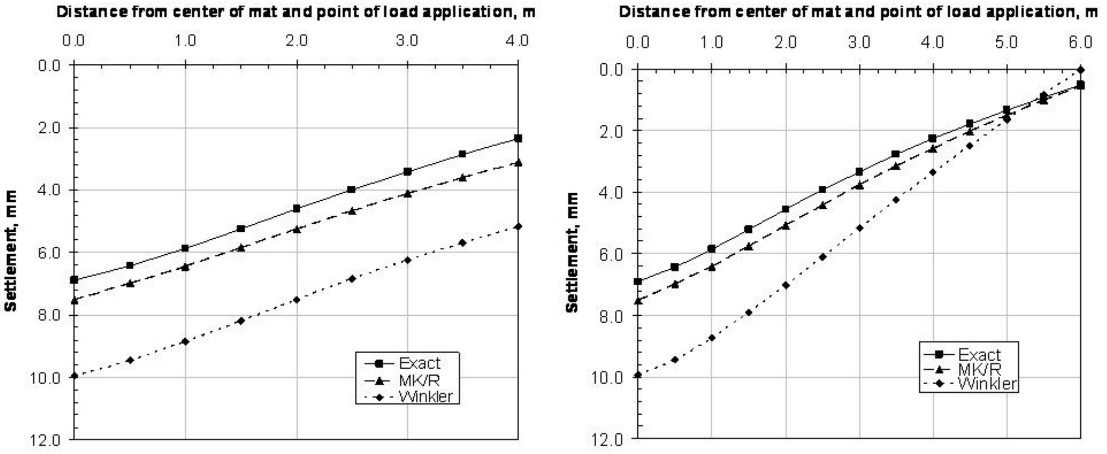


Figure 3. Foundation Example: Problem Details.



Settlements Along X-Axis

Settlements Along Y-Axis

Figure 4. Foundation Example: Calculated Results.

- a finite-element continuum. This provided 'exact' baseline results against which the results from subgrade models were compared.
- MK-R subgrade model.
- Winkler subgrade model.

All assumed and calculated physical and model parameters are shown in Fig. 5. The subgrade was assumed to be 1.376 metres thick (= *H* in Fig. 2) based on using the methodology outlined in Horvath and Colasanti (2009). The in-plane geosynthetic stiffnesses were chosen arbitrarily but to represent a range of values that might be

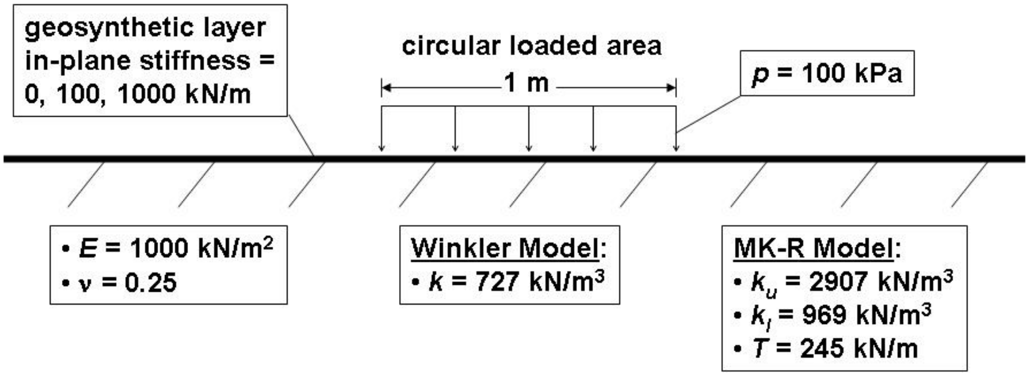


Figure 5. Geosynthetics Example: Problem Details.

encountered using actual materials and products in practice based on information found in Koerner (1998).

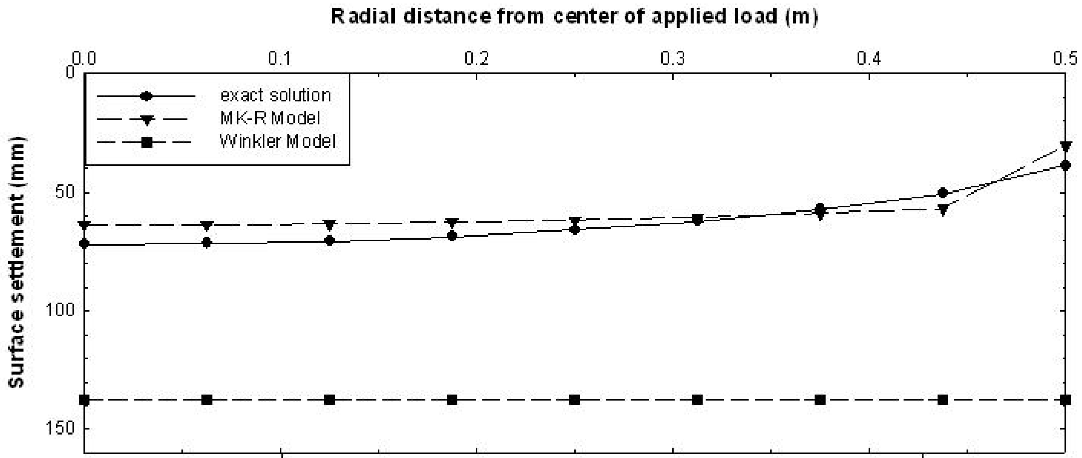
Figs. 6a-c show the calculated results for surface settlements within the loaded area, with a separate sub-figure for each of the three assumed geotextile stiffnesses. The excellent agreement between the MK-R Model and the baseline 'exact' solution obtained using a finite-element analysis of an elastic continuum is apparent as is the superiority, both qualitatively in terms of the shape of the settlement 'bowl' and quantitatively in terms of settlement magnitudes, of the results obtained using the MK-R Model compared to those obtained using the Winkler Model.

Additional Observations

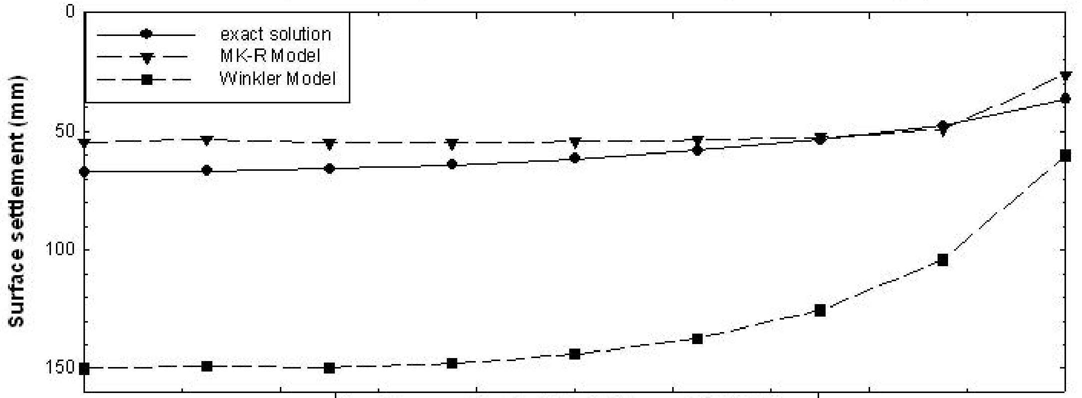
An item of interest in geosynthetics applications such as illustrated in this paper that may not be readily apparent is that the MK-R Model provides insight into not only how geosynthetic tensile reinforcement acts and provides improved subgrade stiffness in the type of problem typified by Fig. 5, i.e. loading perpendicular to the plane of the geosynthetic layer(s), but also allows quantification of this improvement.

Recall that the inherent stiffness-related shearing resistance of a subgrade in a direction parallel to applied loading (vertical for the two example problems in this paper) can be visualized within the context of the MK-R Model as a fictitious constant-tension membrane oriented perpendicular to the direction of loading as shown in Fig. 1. It is this abstract mechanical element that, for the MK-R Model, provides the spring coupling missing from Winkler's Hypothesis. Specifically, it is the differential out-of-plane displacement (deformation) of this fictitious membrane in a direction transverse to its plane under applied load that provides the force between subgrade springs that models the inherent stiffness-based shearing resistance in actual subgrade materials.

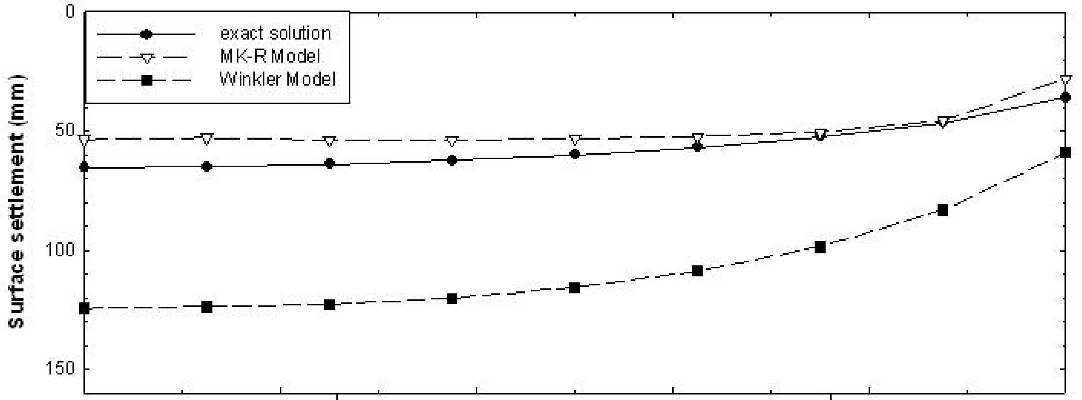
As can be seen in Eqs. 3c and 4c, the magnitude, T, of the tension in this fictitious membrane is linearly proportional to the equivalent isotropic, homogeneous shear modulus, G, and, therefore, the shear stiffness of the actual subgrade materials. Consequently, the actual in-plane tension that develops within a sheet or layer of geosynthetic tensile reinforcement as a result of its transverse differential displacement can be interpreted as being numerically additive to the fictitious, pseudo-membrane tension that reflects the subgrade's inherent shear stiffness. As a



a. No Geosynthetic Reinforcement (Baseline Case)



b. Geosynthetic Modulus = 100 kN/m



c. Geosynthetic Modulus = 1000 kN/m

Figure 6. Geosynthetics Example: Calculated Results.

result, a layer of geosynthetic tensile reinforcement can be visualized as adding to a subgrade's inherent shear stiffness but only if the geosynthetic displaces <u>differentially</u> in a direction transverse to its plane. This is because the shear forces that resist loads of the type shown in Fig. 5 only develop as a result of <u>differential</u>, not absolute, out-of-plane displacement of the geosynthetic. And, of course, the stiffer the geosynthetic the greater the vertical resistance (force) and concomitant contribution to vertical stiffness of the soil-geosynthetic system it will provide for a given magnitude of differential out-of-plane displacement. Finally, note that the magnitude of the in-plane force in the geosynthetic layer(s) is linearly proportional to the slope of the deflected shape of the geosynthetic. With reference to Fig. 6 it is clear that this will occur at the edge of the loaded area.

CONCLUSIONS

The MK-R hybrid subgrade model presented in this paper is believed to be the long-awaited advanced subgrade model that is a significant theoretical improvement over Winkler's Hypothesis as well as straightforward and rational to implement and use within the capabilities of commercially available structural analysis software as well as being easily incorporated within specialized application-specific software. Consequently, the MK-R Model has attraction for both routine use in practice as well as research.

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Note: References marked with a "@" can be downloaded from *www.jshce.com*.

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